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ECONOMIES OF SCALE IN THE SECONDARY EDUCATION SECTOR IN THE PROVINCE OF ONTARIO

by

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Submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy

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The University of Western Ontario
London, Canada
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ABSTRACT

The purpose of this study is to ascertain the existence or non-existence of economies of scale in the secondary school education sector in the Province of Ontario in 1964 and to relate these findings to present changes in the structure of the educational process. In order that a proper analysis may be carried out this sector is divided into two basic sub-sectors, the public, and the private and separate. Further subdivisions are also made within the public sub-sector so that (a) the cost structure of academic and commercial & vocational programmes and (b) the cost structure of schools and school boards, can be examined separately.

The cost curves for each of the functional areas are estimated using cross-section data from all sub-sectors for the year 1964. A stepwise ordinary least squares programme is used as the estimation technique.

Two school quality indices are constructed and used in the study. The first estimates school quality by the quality of students produced--student quality is assessed using tests administered by the Ontario College of Education over a four year period. The second index measures school quality by the quality of inputs in each school, e.g., teacher experience.

The results indicate that, in general, schools exhibit either constant or falling costs per pupil. Thus, there are constant or increasing returns to scale. However, school boards generally exhibit areas of both economies and diseconomies of scale. Further, the use of dummy variables yields two interesting results: (a) the cost per student for school boards operating one school only is significantly lower than that for school boards operating more than one school; (b) the cost per student is higher in the private and separate sub-sector than it is in the public sub-sector.

An attempt is made to estimate the effects of the recent consolidation of school boards in the public sub-sector in the Province of Ontario. The raw data and data weighted by one quality index predict substantially higher costs per student. However, when the data is weighted by the second quality index the result is a prediction of lower costs per student.

PREFACE

As is normal in a study of this type, kudos have been earned by many people. On the technical side I am first indebted to my original thesis committee Professors R. J. Wonnacott, C. D. Hodgins and S. B. Gupta and to Professors K. H. Burley and R. W. Baguley who filled in when two of the original members left on leaves. Computational work was aided greatly by Mr. R. Hartwick of the University of Western Ontario Computing Centre and Miss M. Bowman of the Ontario Institute for Studies in Education. Finally, Mr. R. K. Fletcher of the Grants Section of the Ontario Department of Education extended every possible assistance during the weeks of data collection in Toronto.

Financially, both the Woodrow Wilson Foundation and the Canada Council have helped to sustain the author and kin. Their assistance is gratefully acknowledged at this time.

A special vote of thanks is due to Dean G. L. Reuber who has maintained an interest in the author's

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Finally to my wife Joyce goes the greatest kudo. She has provided financial, clerical and research assistance throughout my graduate studies. Even moreso, however, the love and encouragement which she has continually proffered have provided the vital type of sustenance available from no other source.

D.A.D.

London, Ontario
August, 1969

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CHAPTER I

INTRODUCTION

As noted by Alfred Marshall, the effect of organization on efficiency has been a subject for study by social scientists from the time of Plato.[1] More specifically, economists have long contemplated the problem of economies and diseconomies of scale. However, only in the last two to three decades has a serious attempt been made to quantify the existence of scale economies in specific industries. Most of this effort has been directed towards the primary and secondary sectors. Only recently have services in the public domain (waterworks, police protection, fire protection, etc.) been subject to rigorous statistical analysis.

Education, one of the largest[2] and fastest growing of the public sectors has been no exception

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1 A. Marshall, Principles of Economics, Ninth Edition, Volume 1. New York, The MacMillan Company, 1961, p. 240.

2 For the fiscal year 1969-70 expenditures on education comprise 43% of the Government of Ontario's budget. Source: The Globe and Mail, Toronto, March 5, 1969.

to this lack of study. The bulk of the major economic research in this area began in the early 1960's. Although the main thrust of this research has been in the analysis of human capital (the product of education) some attempts have also been made to measure economies of scale both in schools and in school boards (the structure in which this product is produced.)

Attempts to measure economies of scale in education have been based upon two main questions: first, is there an optimal size of school? Secondly, is there an optimal size of school board? If sufficient data are available a third problem may also be analysed: the relative efficiencies of the private and public portions of the education sector from a cost standpoint. Although a great deal of debate has been generated on this latter subject,[3] only a study by Koos[4] has attempted to quantitatively analyse the two systems.

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[3] Classical liberals have argued for private provision of education. The origins of this debate are recounted in: E. G. West, "Private Versus Public Education: A Classical Economic Dispute," Journal of Political Economy, October, 1964, pp. 465-475. Friedman gives a current viewpoint in: Milton Friedman, Capitalism and Freedom, Chicago, The University of Chicago Press, 1962, pp. 85-107.

[4] L. V. Koos, Private and Public Secondary Education--A Comparative Study, Chicago, The University of Chicago Press, 1931.

The object of this study is to analyse the secondary school[5] education sector in the Province of Ontario as it existed in 1964,[6] in an attempt to obtain answers to the three problems posed above. The choice of Ontario as the study area is particularly appropriate because the provincial Department of Education has been in the process of enlarging school districts. The stated purpose of this procedure has been "to create educational jurisdictions capable of extending equal educational opportunity to the boys and girls of Ontario."[7] Counties have been designated

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5 Secondary schools provide education in Grades 9-13. They have been chosen for this study for two reasons. First, the data problem is more manageable. In 1964, only 257 secondary school boards existed as compared to 2,289 elementary school boards. (The figure 257 is the one presented in the Ontario Department of Education, Report of the Minister, 1964, Toronto, 1965, p. S-2. It differs from the total of 261 which the author has tabulated from individual financial returns. No reason has been found for this difference.) Second, the private versus public debate takes on significance only at the secondary level since even classical liberals allow that the state should provide basic elementary education.

6 The year 1964 has been chosen as the focal point for the study because it is the final year in which students' exam results were recorded by the Ontario Institute for Studies in Education. These results, in turn, form the cornerstone of one of the quality indices utilized in this study.

7 Covering letter attached to Ontario Department of Education, The Reorganization of School Jurisdictions in the Province of Ontario, Toronto, 1968.

as the new areas of school board jurisdiction but no studies appear to have been undertaken to determine whether the new size distribution is optimal for Ontario from an economies of scale aspect.

The study is divided into four parts. In Chapter II the methods of cost analysis are discussed and the results of previous pertinent studies are presented. Chapter III contains a discussion of measures of educational quality followed by the development of quality indices. In Chapter IV the cost and quality data are married up and the resulting cost characteristics are analysed. Chapter V provides a summary of results and conclusions as well as suggestions for further research in the area.

CHAPTER II

COST STUDIES[1]

Introduction

Before analysing the educational sector from the cost point of view it is necessary first to look briefly at the various methods of constructing firm and plant cost curves. The different methods have both benefits and defects which must be recognized before deciding on the one best suited to this particular study.

As a second prerequisite, we must categorize the various types of costs, both direct and indirect, which seem to be related to education. It must be decided which of the costs actually are economic costs of education as opposed to costs which merely apply to services

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[1] An excellent survey of cost studies both from the theoretical and practical points of view is contained in: J. Johnston, Statistical Cost Analysis. New York, McGraw-Hill Book Company, 1963.

rendered to those being educated but not the educational process itself. The dividing line is often vague but it must be drawn. The position of this dividing line will be a prime determinant of the position of the resulting cost curves.

Additional benefit will be derived by reviewing past studies in the field of educational cost analysis. Only by looking at these studies will we be able to get advance notice of some of the pitfalls which exist and paths around them. The methodology used in these studies might also suggest new methods of approach or possible extensions of existing methods.

Methods of Analysis

Firm or plant costs have typically been studied from either time series or cross section data. Only recently has a third method been emphasized, that of survivorship. Each is briefly examined.

The time series approach looks at the financial records of plants or firms over time and analyses the related changes in output and cost. Assuming some change in demand necessitating a change in fixed assets has taken place, a long run cost curve can be constructed.

The shape of the resulting curve reveals the existence and extent of economies or diseconomies of scale. The apparent simplicity of this approach is, however, deceiving because of the problems involved in allowing for changes in prices of factors of production. In addition, for this approach to be valid, it is necessary to assume that firms or plants are at all times producing at their most 'efficient' level, i.e., where Short Run Average Cost equals Long Run Average Cost.

Using the second approach, cross section analysis, it is necessary to analyse the cost and output positions of all firms and plants in an industry at one point in time. Assuming (a) that all firms and plants are operating 'efficiently'; and (b) that there is a range of plant and firm sizes, then a scatter diagram indicating a Long Run Average Cost Curve can be plotted. This method generally avoids the problem of price indices, although some spatial differences may have to be considered and it is more appropriate for an industry "subject to slow technical changes."[2] If this were not

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[2] _Ibid._, p. 30.

so newer plants and firms would be working with a markedly different technology than the older ones; they would, in effect, be observations on a different cost curve. The mixing of observations of new and old plants and firms may thus result in a poor approximation of the current cost curve which, ideally, should give "the best results achievable at various scales of output, given the current state of technology."[3]

One problem arises in cross section analysis in that there is no independent force such as demand changes which can account for the different levels of equilibrium.[4] In fact, if, as Friedman points out, price of output is the same for all firms in a competitive industry and costs are defined as including profits, then the cross section will reveal constant

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3 Ibid., p. 30.

4 A. A. Walters, "Production and Cost Functions," Econometrica, January-April, 1963, p. 44.

average costs.[5] Variations in output amongst firms would then "be due to either 'mistakes' or the existence of specialized resources controlled by the firm."[6] However, Johnston notes that in contrast to perfectly competitive industries, regulated or imperfectly competitive firms owe both their number and relative scale to the spatial distribution of demand. Accordingly, "size variations are no longer mistakes."[7]

Another possible problem in cross section studies is that of the bias created by the "regression fallacy."[8] It is argued that all firms face random variations in output. To the extent that output is below the firm's 'average', costs will appear to be larger than normal

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5 M. Friedman, "Comment," National Bureau of Economic Research, in: Business Concentration and Price Policy. Princeton, N. J., Princeton University Press, 1955, pp. 230-238.

6 Walters, *op. cit.*, p. 44.

7 Johnston, *op. cit.*, p. 187.

8 This "fallacy" is discussed in Johnston, *op. cit.*, pp. 188-194 and in Walters, *op. cit.*, p. 48.

because of fixed cost components. Conversely, for firms producing above 'average', costs will appear to be lower than normal. The result of the bias is to rotate the cost curve in a clockwise manner and produce a more downward sloping curve. Although this "fallacy" is considered to be important in industry studies, it is doubtful if it applies as much to education. Population movements and birth rates are relatively slow moving and predictable so at any one time a school board or school is unlikely to face any great variation in students.

The survivorship principle emphasizes the fact that over time "competition of different sizes of firms sifts out the more efficient enterprises."[9] It avoids the problem of price indices involved with resource valuation. Basically all that is necessary is to classify the firms in an industry by size and calculate the share of output coming from each class over time.[10]

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9 G. J. Stigler, "The Economies of Scale," The Journal of Law and Economics, October, 1958, p. 55.

10 Ibid., p. 56.

If the share of a class should fall it can be considered to be in a relatively inefficient range.

The validity of this principle in a study of economies of scale in education is, however, questionable. The size distribution within the educational sector will in fact be determined largely by spatial demand factors. School boards do not compete with one another; each is a monopolist in it's own market. (This, of course, neglects the role of the independent schools which compete nationally.)

Of the methods available it seems clear the the one best suited to this study is that of cross section analysis. Although spatial price differences do exist in Ontario it is doubtful that they will create as much difficulty as would the construction of temporal price indices. In addition, the rapid decline in secondary school boards over the past years would render the consistent identification of production units very difficult.[11]

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11 Secondary school boards declined from 306 in 1955 to 237 in 1967. Source: Ontario Department of Education, The Reorganization of School Jurisdictions in the Province of Ontario, Toronto, 1968, Appendix 'A'.

Relevant Costs

Costs of education can be divided into two main categories, direct and indirect.[12] Each of these categories is analysed to determine which costs must be included in any analysis of the economies of scale in education.

The main sources of direct costs are operational expenditures; it is generally agreed that these should be included in a summation of economic costs. Examples of this type of cost are teachers' salaries, plant operation, etc. It is also generally agreed that expenditures on items such as transportation and school lunches should be excluded. With regard to the former, Riew notes that "the per pupil costs of transportation often vary more with population density, and the distance a school bus has to travel than with the size of school which may or may not reflect the population density."[13]

Significant cost items which are not dealt with

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12 R. Blitz, "The Nation's Educational Outlay," in: Economics of Higher Education, S. J. Mushkin, Ed., United States Department of Health, Education and Welfare, Bulletin, 1962, no. 5, Washington, 1962, p. 150.

13 J. Riew, "Economies of Scale in High School Operation," Review of Economics and Statistics, August, 1966, p. 283.

in any standardized way are capital outlays, debt service and depreciation. Capital outlays for land, buildings and equipment are indeed valid costs of education. Since they occur at rather sporadic intervals, however, a cross section analysis will pick up only those expenditures which occur in the survey year. Those schools or school boards which have capital outlays in the period will appear to have higher costs even after these costs are "annualized."[14] Debt service also appears to be a valid cost and it avoids most of the bias involved in capital outlays since it is spread out over a much longer period of time.

In an effort to avoid the bias which would be created by the exclusion of capital outlays, depreciation charges on the present value of buildings and equipment are included in the total cost summation. Unfortunately, depreciation is not allowed for by operating units in the Ontario Secondary School sector so it must be imputed. Depreciation is calculated at a

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[14] This term is used by Edding and means basically the application of a certain portion of capital outlays to future years. It is more fully described in: F. Edding, Methods of Analysing Educational Outlay, Paris, UNESCO, 1966, p. 15.

rate of 2.44% per year and is applied to the present value of all schools' buildings and equipment.[15]

A final direct cost to be considered is property and sales taxes. Secondary school units do not pay these taxes but they must be calculated as government supplies services and facilities which are important to the educational process (e.g., public libraries). Property taxes based on the present value of land, buildings and equipment will

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15 This rather unconventional figure is arrived at as follows. First a standard rate of 2% for buildings and 5% for equipment has been adopted. This follows the procedure found in: R. W. Goldsmith, "A Perpetual Inventory of National Wealth," Studies in Income and Wealth, Volume 14. New York, National Bureau of Economic Research, 1951, pp. 22-23. Next we compute the ratio of furniture and equipment to buildings, at original cost, for all reporting school boards. This is necessary because the present value of these items is shown as a combined total only. Weighting the resulting ratio by the relevant rates, we finally arrive at the value of 2.44%.

be imputed.[16] In addition, sales tax at a rate of 3% will be computed on instructional supply purchases.[17]

The indirect cost of education is the income foregone by students and imputing this cost would create some major problems.[18] However, since this

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16 The basic mill rates used are the commercial rates which were prevalent in the areas in which the school boards were located in 1964. The source of these rates is: Province of Ontario, Department of Municipal Affairs, Analysis of the Mill Rate on the Residential Assessment of a Public School Supporter, 1964, unpublished. The basic rates have been reduced by the percentage of total tax revenue which goes to education.

The possibility of a bias arises here in that large school boards might exist in areas with high mill rates (or vice versa). To test for this bias, school board size (measured by average daily attendance) has been correlated with the mill rate for 206 school boards. The resulting correlation coefficient is -.174 which is statistically insignificant at the .01 level of significance.

17 The sales tax in Ontario in 1964 was at the rate of 3%.

18 The main problems are as follows. (a) Can the wages of 'drop-outs' be taken as proxies for the opportunity costs of students still in school or would these students actually be worth more because of accumulated human capital? (b) Should we take into consideration the fact that if all school children entered the market the marginal productivity and therefore opportunity cost would decline? (c) Should we consider the possibility that vacancies might not be available, a condition which would cause opportunity cost to be zero?

cost should not, in general, vary between schools and school boards our analysis is not hurt by omitting it. The relative position of economies and diseconomies will not change because of this deletion, only the height of the resulting cost curve will be effected.

Given the above considerations, the following costs have been used to calculate Total Cost for our study.

(a) Administrative costs less membership fees in educational associations.[19]

(b) Plant operation and maintenance costs.

(c) Instructional salaries.

(d) Instructional supplies.

(e) Interest on temporary borrowing.

(f) Long term interest charges.

(g) Depreciation.

(h) Property Tax.

(i) Sales Tax.

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19 Membership fees are omitted since they do not directly influence the educational product.

Previous Studies

In a cross section study of 577 school districts in nine states, Hanson[20] uses two variables. The independent variable is school district size while the dependent variable is "a unit cost residual from each district, which was obtained by adjusting current expenditures per pupil for the influence of certain characteristics of the adult population upon expenditure levels."[21]

For each district, expenditure on education is predicted using the population characteristics. The resulting residual between the predicted and actual values is then regressed on school district size.

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[20] N. W. Hanson, "Economy of Scale as a Cost Factor in Financing Public Schools," National Tax Journal, March, 1964, pp. 92-95.

[21] Ibid., p. 93. The characteristics are:

(a) fully equalized property value per pupil
(b) median family income
(c) percent of homes owner occupied
(d) median years schooling of the adult population.
(e) percent of the working force unemployed
(f) percent of the population non-white
(g) percent of the population living in rural areas
(h) percent of the school age population enrolled in independent schools

The aim of this procedure is to isolate the effect of size on cost and when it is carried out a U-shaped cost curve is the general rule in six of the nine states. In the three remaining states, continuing economies are present. The median optimum size is 50,000 pupils.[22]

Hanson's study, though original in approach, has three major shortcomings. First, it fails to allow for quality differences between schools. Next, by pooling data from elementary and secondary schools, an aggregate figure results which conceals specific information about either type of school.[23] Finally, by using a residual in an attempt to test for economies, a bias will result in the coefficients associated with the independent variable if, as may be expected, there

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22 The median is calculated from the results obtained in each of the nine states.

23 That these types of schools are basically different is shown in the fact that in Ontario in 1963 the cost per elementary pupil was $326.98 and for each secondary pupil it was $664.26. Source: Ontario Department of Education, Report of the Minister, 1964. Toronto, 1964, p. S-158.

is correlation between size and the population characteristics.[24]

Using another approach, Hirsch[25] studies the thirty school districts in the St. Louis City-County area. In an attempt to explain the determinants of expenditure he runs a multiple regression using eight variables. They are (a) average current expenditure plus debt service (the dependent variable); (b) number of pupils in Average Daily Attendance; (c) variable (b) squared; (d) high school pupils as a percent of all pupils; (e) number of pupils per square mile; (f) percent increase in public school pupils in Average Daily Attendance, 1951-56; (g) average assessed valuation of real property per pupil in Average Daily Attendance; and (h) index of quality and scope of public

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[24] This problem is explained in: A. S. Goldberger, Econometric Theory. New York, John Wiley & Sons, Inc., 1964, pp. 194-197.

[25] W. Z. Hirsch, "Determinants of Public Education Expenditures," National Tax Journal, March, 1960, pp. 29-40.

education.[26]

Hirsch pools cross section data from two non-coterminus periods, 1951-52, and 1954-55 and finds only variables (d), (g), and (h) are statistically significant. Given the statistical insignificance of variables (b) and (c) he concludes that there is an absence of significant economies of scale in the school districts of the St. Louis City-County area.[27]

This study has the same weakness as that of Hanson's in that the cost figures for the primary and secondary education are not separated out. In addition, the use, in this study, of data from two close but separated periods raises the question of serial correlation.[28] Hirsch unfortunately does not

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[26] This index is composed of the following six equally-weighted sub-indices:
  (1) Number of teachers per 100 pupils.
  (2) Number of college hours of average teacher.
  (3) Average teacher salary.
  (4) Percent of teachers with more than ten years experience.
  (5) Number of high school credit units.
  (6) Percent of high school seniors entering college.

[27] Hirsch, op. cit., p. 36.

[28] The author might have used a generalized regression model to negate the possibility of this problem. The procedure is discussed in Goldberger, op. cit., pp. 231-235.

deal with this difficulty in detail. Instead, he assumes that "since expenditures and their determinants increased over time at somewhat different rates in the different school districts, the cross section data . . . was devoid of any major serial correlation."[29]

Another cross section study is that carried out by the Netherlands Central Bureau of Statistics as part of the Bureau's Statistical Investigations on Education and Leisure.[30] In this study a regression is run with total cost per annum (excluding investment cost) as the dependent variable and number of pupils as the independent variable. Given the data from 330 schools the regression produces an L-shaped average cost curve with the horizontal portion beginning in the range of 500-600 pupils. Again, this study does not take quality into consideration. Also, tests are not carried out to determine whether other structural forms produce better fits.

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[29] Hirsch, op. cit., p. 35.

[30] Netherlands Central Bureau of Statistics, Statistical Investigations on Education and Leisure (2)--Size and Cost of Secondary Grammer Schools, The Hague, 1965.

In another cross section study, Riew[31] concentrates on a sample of senior high schools (grades 9-12 or 10-12) in Wisconsin. In an attempt to equalize quality Riew first accepts into his sample only those schools which are accredited by the North Central Association and then eliminates those accredited schools which have an average teacher salary of more than $6,500.

On the cost side he excludes capital outlays, debt service, transport and auxiliary services (lunches, recreation, etc.). Dividing the resulting cost by the number of pupils in Average Daily Attendance, this value becomes the dependent variable. The independent variables in the least squares multiple regressions are (a) enrolment; (b) average teacher salary; (c) number of credit units offered; (d) average number of courses taught per teacher; (e) change in enrolment between 1957 and 1960; and (f) percent of classrooms built after 1950.

Variables (b), (c), and (d) are included as

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31 Riew, op. cit., pp. 280-287.

quality variables which represent "teacher qualifications, breadth of curriculum and degree of specialization in instruction."[32] Variable (e) is included to allow for change in demand, and variable (f) to allow for variation in the cost of maintenance with the age of capital.

The regression is run with a sample size of 109 schools and variables (a), (a^2), (b), and (e) are found to be statistically significant. The minimum point on the resulting average cost curve is at 1,675 pupils.[33] Thus, this study indicates the presence of substantial economies of scale. However, the results of the study are somewhat weakened by the absence of a specific quality index. In addition, it would have been more illuminating had the analysis been extended to analyse economies of scale within school districts.

A final cross section study is that of Ellis.[34] The sample used is 76 Kansas school districts and Ellis

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32 Ibid., p. 285.

33 The largest school in the sample has an enrolment of 2,400.

34 J. W. Ellis, An Investigation of the Cost Function for Kansas School Districts, 1965-66. M. A. Thesis, Department of Economics, Wichita State University, June, 1967.

states that "a student in one district . . . is assumed to be receiving the same general type and quality of education as a student in any other district."[35] No significant economies of scale are discovered. The only statistically significant elements produced by the regression analysis are a constant term, the number of students per teacher and a dummy variable which indicates that the school district offers elementary education. This study is subject to two of the same criticisms attributed to that of Hanson. First, there is no quality measure and second there is no disaggregation into primary and secondary sectors.

Summary

In this chapter we have concluded that a cross-section study is the best means of analysis for our purposes. Although a problem of spatial price differentials may arise using this method, this problem is a minor one compared to that of the construction of price

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[35] Ibid., p. 16.

indices for all factors of production which would be necessary in time series analysis. The third possible method of analysis, survivorship, has been eliminated from consideration due to the existence of local monopolies in education, a fact which negates the survivorship premise of competition among producing units.

Following a brief analysis of the costs of education, we have excluded indirect costs from our calculation of the total cost of education because of the problems involved in quantifying this type of cost. The direct costs which comprise the total cost figure in this study are (a) administrative costs; (b) plant operation and maintenance costs; (c) instructional salaries; (d) instructional supplies; (e) interest charges; (f) depreciation; (g) property tax; and (h) sales tax.

Finally this chapter has presented a summary of other studies of economies of scale in education. These studies were usually lacking in two aspects. They generally allowed for neither the quality of education nor the difference in cost structures between the primary and secondary school sectors. Accordingly, in this study an attempt will be made to avoid these criticisms.

CHAPTER III

QUALITY OF EDUCATION

Introduction

The construction of realistic cost curves in order to (a) check for economies of scale; or (b) compare costs between different systems requires first that a correction be made for the "quality" of student produced by each secondary school and secondary school board in Ontario.[1] Failure to make such a correction could lead to an incorrect assessment of the existence of economies. For example, School A may have an expenditure per pupil which is twice as high as that of School B. However, if the average quality of the students produced in School A is three times as great as that of students produced in School B, it would be incorrect to state that the true costs in

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[1] The word quality is unfortunately euphamistic. Opinions as to what is a high quality pupil will vary from person to person or locality to locality. The measures discussed in this chapter are general and encompass broad concepts of the definition of quality.

School B are lower than those in School A. Clearly the corrected costs in School A are lower.

The analysis of educational quality has a long history beginning, perhaps, with the preliminary work on the problem by Ayres[2] and Cooke.[3] In general, there are two basic types of quality measures, objective measures and subjective measures. The former, in turn, can be approached from the 'product' point of view or the 'process' point of view. It is our aim in this chapter to analyse the types of measures and construct those considered most suitable for Ontario.

Objective Quality Measures

A 'product' objective measure typically measures quality by the level of student attainment on various types of achievement tests. Many problems arise with the use of this type of measure. First, the goals of

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[2] L. P. Ayres, Laggards in Our Schools. New York, Russell Sage Foundation, 1909.

[3] M. E. Cooke, Academic and Industrial Efficiency. New York, The Carnegie Foundation for the Advancement of Teaching, 1910.

different communities or systems may differ and no one test or set of tests can cover the objectives of all the communities and systems at once. Second, "tests cannot and do not cover _all_ aspects of learning that are worth-while in . . . subject areas."[4] Third, performance depends to a large but unmeasured extent upon the pupil's socio-economic background. Finally, the results of such tests depend upon the individuals' "frame of mind, his motivation, his health, and many other chance factors and not merely on the extent of his learning."[5]

Probably the most widely known 'product' measure is that which has been constructed by the Quality Measurement Project of the New York State Education Department.[6] This project tests 70,000 students in 100

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[4] J. A. Kershaw and R. N. McKean, _Systems Analysis and Education_. RAND Corporation Research Memorandum, RM-2473-FF, October, 1959, p. 10.

[5] _Ibid_., p. 11.

[6] New York State Education Department Quality Measurement Project, _Procedures in School Quality Evaluation_. New York State Education Department, Albany, 1961.

school systems over a four-year period.[7] The testing instruments are the Iowa Tests of Basic skills or the Iowa Tests of Educational Development.[8] Another, more recent test, is that carried out using "Project TALENT" data on 206 high schools.[9]

Because of the cost of sampling and measurement problems involved in 'product' measures, most authors have exhibited a preference for 'process' measures. The process measures typically focus on (a) the quality of inputs; or (b) the adaptability of school systems. Examples of variables which have been put forward as good quality of input measures are (a) teacher/student ratio; (b) teacher turnover; (c) teacher experience; and (d) teacher education. A fairly exhaustive com-

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7 W. D. Firman, "The Relationship of Cost to Quality in Education," in: Long Range Planning in School Finance. Finance Conference, based upon the Proceedings of the Sixth National School Finance Conference, St. Louis, Mo., April 7-9, 1963, p. 102.

8 Procedures in School Quality Evaluation, op. cit., p. 26.

9 Some results are reported in: J. A. Thomas, "Efficiency in Education: An Empirical Study," Administrator's Notebook, October, 1962.

pilation of measures of this type is attached as Appendix A.

The best known measure of adaptability is "The Growing Edge."[10] This device measures the degree to which schools have changed their teaching techniques and curricula in recognition of new knowledge from the fields of psychology and/or educational research.[11] Unfortunately, this type of index is very expensive to create and accordingly will not be used in this study.

Subjective Quality Measures

As their name implies, subjective measures are not easily constructed.[12] Assessors must go into each school in the test area and give ratings on such measures as "Discovery and Development of Special Aptitudes," "Development of Gross Behavior Patterns" and "Amount of Consideration of the Individual." It is very difficult to obtain consistent estimates of these measures unless

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[10] P. R. Mort, W. S. Vincent and C. A. Newell, The Growing Edge. New York, Metropolitan School Study Council, 1946.

[11] Procedures in School Quality Evaluation, op. cit., p. 22.

[12] A good brief discussion is contained in: W. E. Barron, "Measurement of Educational Productivity" in: Theory and Practice of School Finance, Gauerke and Childress (Eds.), Chicago, Rand-McNally, 1967, pp. 293-96.

one person does all the assessing. Further, the employment of the teams of researchers needed for the development of this type of measure over a broad area would prove tobe very costly. Owing to these problems an attempt has not been made to construct a subjective quality measure for use in this study.

Construction of Quality Indices

Our purpose is to construct cross section quality indices. Individuals have constructed these indices to measure differentials in the quality of such diverse items as mental patient care, farm housing, religious work in diocese and the agricultural value of soils. In the field of education, initial efforts to quantify state quality differentials have been made by Ayres[13] and Philips.[14] A more recent work in this area is the educational index developed by Hirsch as described in Chapter II.

When constructing indices of this type three

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13 L. P. Ayres, An Index Number of State School Systems, Russell Sage Foundation, New York, 1920.

14 F. M. Philips, Educational Ranking of States by Two Methods. Bruce Publishing Company, Milwaukee, 1925.

basic methods can be adopted. In Method I all observations for each variable to be included in the index are ranked. The ranks obtained by each school (in this case) for each variable are then summed and divided by the number of variables in the index.[15] In Method II the value of all observations of a variable is divided by the mean of that variable. To get each school's

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[15] In algebraic terms:

$$\text{Method I index} = \frac{\Sigma X}{N}$$

where X = rank in variable
N = number of variables

$$\text{Method II index} = \frac{\Sigma \frac{X}{U}}{N} \qquad \text{and}$$

$$\text{Method III index} = \frac{\Sigma Z}{N}$$

where X = value of a given observation for a specific variable
U = mean of all observations for a specific variable
N = number of variables
σ = standard deviation of observations for a given variable

$$Z = \frac{X-U}{\sigma}$$

index, we sum its corrected observation for each variable and divide by the number of variables in the index. In Method III each observation is standardized and then summed in the same manner as Method II.

McMillan[16] compares the results obtained using these three methods to calculate Farm Housing Indices and finds the indices to be highly correlated. He also points out that although the first two indices are simple to construct there is a problem, particularly with Method II, in that those items which possess greater variability receive larger weights. Method III eliminates this weakness by dividing by the standard deviation and for this reason will be used in the construction of our quality indices.

Our 'product' quality measure, Index A is based upon scores achieved in tests administered by the Carnegie Foundation to students registered in Grade 9 in all sectors of the Ontario Secondary School System

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16 R. T. McMillan, "Comparison of Farm Housing Indexes for Oklahoma," Social Forces, October '45 - May '46.

in 1959.[17] The 90, 719 students initially wrote the CAAT, CEAT and CMAT. The following year the same students (minus 'drop-outs') wrote the CTGF, CATF, CATE, and CATM. In Grade 11 they wrote the CPTO and CGT and in Grade 12 the remainder of the tests were written. These tests are fully titled in Appendix B.

Rather than measuring quality by absolute scores, Index A is constructed by looking at changes in average scores in tests which measure the same abilities.[18] The tests which are paired are indicated in Table I. (i.e., X_1 is the change in scores achieved in CMTI and SCHAptVerbTot).

Thus, to obtain a school's quality rating using Index A, we compute the standardized value of the change

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17 Information about the tests and the resulting data bank which has been formed is contained in: "Carnegie Human Resources Data Bank," Pamphlet No. 1, Carnegie Study of Identification and Utilization of Talents in High School and College, The Ontario Institute for Studies in Education, Toronto, undated.

18 Changes in scores rather than the absolute value of scores are utilized to avoid some of the bias introduced by socio-cultural background.

TABLE I

TESTS PAIRED FOR USE IN OUTPUT QUALITY INDEX

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
CAAT I	1																
CAAT II	2																
CAAT III	3																
CEAT I	4																
CEAT II	5																
CEAT III	6																
CMAT I	7																
CMAT II	8																
CMAT III	9																
CTGI	10																
CATF	11																
CATE	12				X_7	X_9	X_{10}										
CATM	13		X_3					X_{11}	X_{15}	X_{19}							
CPhyTO	14		X_4					X_{12}	X_{16}	X_{20}							
CGeomT	15		X_5					X_{13}	X_{17}	X_{21}				X_{26}			
SCHAptTest Verb I	16																
SCHAptTest Verb II	17																
SchAptTest Verb Tot	18	X_1			X_8							X_{24}					
SCHAptTest Mat	19		X_6					X_{14}	X_{18}	X_{22}				X_{27}	X_{28}	X_{29}	
Eng TESU	20	X_2										X_{25}					X_{30}
Fr ATF	21										X_{23}						

in the average mark obtained in the school in each of the paired tests, sum the values and divide by the number of paired tests (i.e., Method III).[19] To avoid problems involving student movement between schools, only the scores of those students remaining in one school throughout the entire four year testing period are taken into consideration when evaluating a school's performance.

We expect a positive correlation between the pairings, i.e., a 'good' school should show a consistent improvement. However, the matrix of correlations shown at Table II indicates that variables 7, 9, 27, 28, 29 and 30 fail to yield a general positive correlation and there is no simple explanation for this fact. Three of those pairings are in Mathematics and Science and three in English so the cause of the perverse behavior does not lie in any one subject area. However, despite the lack of an explanation for the negative values, it is apparent that something must be done about these six pairings and our method will be simply to delete these variables. Thus, the construction of Index A will be based on the

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19 A further normalization has been carried out to ensure that the mean of this Index is 1 and all values are positive. This has been done by first shifting the axis so that all Index values are positive and then dividing by the mean of the distribution.

TABLE II

CORRELATIONS OF PAIRED TESTS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1																														
2	.88																													
3	.16	.20																												
4	.73	.66	.41																											
5	.58	.46	.55	.69																										
6	.59	.61	.12	.41	.33																									
7	[illegible]	-.34	[illegible]	[illegible]	-.39	-.09																								
8	[illegible]	.71	.12	.52	.44	.68	-.01																							
9	-.59	-.39	-.11	-.54	-.62	-.11	-.96	-.03																						
10	.27	.12	-.04	-.02	-.08	-.16	.55	.28	.46																					
11	.12	.15	.94	.35	.51	.06	.10	.24	.09	.16																				
12	.68	.57	.35	.92	.63	.29	-.32	.62	-.33	.23	.42																			
13	[illegible]	.19	.44	.59	.87	.20	-.16	.58	-.16	.28	.36	.73																		
14	[illegible]	.24	.01	.15	.15	.58	[illegible]	.76	.41	.41	.23	.39	.67																	
15	.13	.20	.92	.34	.49	.08	.15	.25	.13	.21	.98	.40	.53	.21																
16	[illegible]	[illegible]	[illegible]	.91	.61	[illegible]	-.26	.61	-.28	.30	.41	.94	.70	.36	.43															
17	[illegible]	.41	.44	[illegible]	.84	.19	-.07	.58	-.08	.36	.56	[illegible]	.98	.45	.57	.71														
18	[illegible]	[illegible]	-.22	.11	.10	.55	.45	.73	.49	.51	.20	.32	.40	.94	.24	.36	.46													
19	[illegible]	.23	[illegible]	.32	.48	.06	.13	.17	.12	.14	.98	.36	.50	.17	.98	.37	.52	.18												
20	[illegible]	[illegible]	[illegible]	.95	[illegible]	.32	-.34	.41	-.36	.21	.41	.98	.69	.32	.40	.98	.69	.30	.37											
21	[illegible]	.42	.48	.62	.91	.23	-.17	.56	-.15	.24	.56	.71	.98	.39	.56	.71	.98	.37	.53	.71										
22	[illegible]	.34	.22	.14	.13	.64	.41	.74	.45	.38	.19	.31	.36	.92	.23	.34	.40	.95	.20	.32	.36									
23	.24	.24	.31	[illegible]	[illegible]	[illegible]	.36	.42	[illegible]	-.11	.32	.44	.42	.37	.33	.45	.43	.38	.33	.44	.45	.41								
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.52	[illegible]	.74	[illegible]	-.12	.11	.69	.55	.33	.09	.65	.51	.25	.05	.69	.55	.27	.25							
25	[illegible]	.92	[illegible]	.61	[illegible]	[illegible]	[illegible]	.74	-.39	-.21	.16	.57	.42	.40	.16	.54	.40	.34	.12	.58	.43	.41	.36	.85						
26	[illegible]	[illegible]	[illegible]	.24	.38	.27	-.31	.35	-.31	-.03	-.51	.25	.38	.22	-.53	.22	.35	.16	-.56	.24	.37	.15	-.07	.47	.33					
27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.30	[illegible]	.16	.36	.03	-.88	-.24	-.34	.25	-.86	-.24	-.35	.25	-.88	-.24	-.37	.26	-.09	-.07	[illegible]	[illegible]				
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-.22	.52	-.06	-.36	-.85	-.53	.14	-.38	-.85	-.51	.17	-.33	-.87	-.55	.18	-.28	-.51	-.35	-.11	.43			
29	[illegible]	[illegible]	[illegible]	[illegible]	-.96	.13	.37	-.13	[illegible]	.01	-.52	-.53	-.81	.13	-.50	-.51	-.79	.17	-.49	-.54	-.89	.19	-.29	-.37	-.21	[illegible]	[illegible]	[illegible]		
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-.17	.71	[illegible]	.75	-.05	-.02	-.59	-.49	-.09	.01	-.56	-.45	-.19	.04	-.58	-.48	.03	.72	-.82	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	

[illegible] = Variable [illegible] minus Variable [illegible]

A_{11} = Variable 13 minus Variable 7
A_{12} = " 14 " " 7
A_{13} = " 15 " " 7
A_{14} = " 19 " " 7
A_{15} = " [illegible] " " 8
A_{16} = " 14 " " 8
A_{17} = " 15 " " 8
A_{18} = " 19 " " 8
A_{19} = " 13 " " 9
A_{20} = " 14 " " 9

A_{21} = Variable [illegible] minus Variable [illegible]
A_{22} = " 19 " " [illegible]
A_{23} = " 21 " " [illegible]
A_{24} = " [illegible] " " [illegible]
A_{25} = [illegible]
A_{26} = [illegible]
A_{27} = [illegible]
A_{28} = [illegible]
A_{29} = [illegible]
A_{30} = [illegible]

remaining 24 variables so that we may preserve a positively signed homogeneity.

Our 'process' quality index, Index B, is constructed from the input rather than the adaptability point of view. Appendix A indicates that there are many likely measures from which a 'process' quality index can be constructed. However, because of data problems, only nine measures are considered for inclusion into a composite index. The measures are as follows: (a) teacher/student ratio; (b) teacher turnover;[20] (c) average number of teachers with an M.A. degree; (d) average number of fully qualified teachers;[21] (e) average number of teachers with a B. A. degree; (f) average number of basic certificates per teacher; (g) average number of specialist certificates per teacher; (h) average number of teachers with 5 or more year's

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20 This variable indicates the number of teachers remaining in a school over a specific number of years. In our case it is the turnover between 1963 and 1964.

21 There are teachers who are not teaching with a temporary certificate or letter of permission.

experience; (i) average number of teachers with 2 or more year's experience. The components mentioned in the measures are fully defined in Appendix C. The data for number of teachers are compiled from the 1964 Report of the Minister of Education, and the remaining figures are taken from the 1964 "Blue Book."[22]

A matrix of simple correlations between the measures is shown at Table III. These results indicate that all are positively correlated except for Variable (a) [teacher/student ratio] and seven of the remaining eight variables. In this case the correlations are negative. This result is surprising but, since teacher/student ratio is also a proxy for class size, it would seem to lend some credence to the poor rating given this latter variable by the authors indicated under Measure 3 in Appendix A. Raymond, in a recent article,[23]

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22 Government of Ontario, Department of Education, Schools and Teachers in the Province of Ontario - Part II - Secondary Schools Teachers' Colleges, and Technical Institutes, 1964, Queen's Printer, Toronto, 1964.

23 R. Raymond, "Determinants of the Quality of Primary and Secondary Public Education in West Virginia," The Journal of Human Resources, Vol. III, No. 4., pp. 450-470.

TABLE III

CORRELATION OF MEASURES

Measures	a	b	c	d	e	f	g	h	i
a	1.0								
b	-0.121	1.0							
c	-0.023	0.021	1.0						
d	-0.063	0.149	0.426	1.0					
e	-0.127	0.020	0.473	0.458	1.0				
f	-0.103	0.275	0.338	0.530	0.312	1.0			
g	0.139	0.086	0.128	0.229	0.125	0.337	1.0		
h	-0.171	0.145	0.278	0.399	0.309	0.460	0.495	1.0	
i	-0.172	0.232	0.297	0.400	0.278	0.467	0.367	0.699	1.0

Critical Value 95% - 0.164

99% - 0.230

also finds the teacher/student ratio to be insignificant. He postulates that high ratios may be due to under-untilization of teachers in rural areas and the ratio might then, in turn, be a proxy for an inadequately organized school system and could therefore be associated with poor quality.

The general positive correlation between the remaining variables indicates that they might all be considered measures of the same thing (quality). Accordingly, Index B is constructed using method III on these eight variables.[24]

Given the rationale behind their construction it is to be expected that Index A and Index B are positively and significantly correlated. However, an analysis of 303 paired observations[25] yields an insignificant correlation of -.02 between them.

An attempt has been made to make the indices more

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[24] The normalization described in Footnote 19 is also carried out on Index B.

[25] Only 303 schools are utilized because Carnegie Data is lacking for other relevant schools. The schools for which data is lacking come from no particular size class.

comparable by means of factor analysis.[26] Specifically, each component of a factor (i.e., each variable in the relevant quality index) is weighted by it's factor loading which is arrived at by the "Varimax" method.[27] New quality indices are now constructed using the weights. These new indices are then compared. This analysis is simplified by the fact that when a factor analysis using both the "Principal Axis" and "Varimax" methods is carried out on Index B, only one factor falls out. On the other hand, Index A yields five factors (A, B, C, D, and E). The correlations between the weighted indices are shown at Table IV. Unfortunately,

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26 The technique of factor analysis was pioneered by the psychologists Charles Spearman and Louis L. Thurston. Simply stated, the purpose of factor analysis is to examine a number of variables to see which sub-groups of variables are correlated to some 'factor'. Once the sub-groupings are completed it is then necessary to identify what each of the 'factors' are. In our case it was assumed that one of the resulting factors might be quality. A good description of factor analysis from an economic point of view is contained in: I. Adelman and C. T. Morris, Society, Politics and Economic Development, A Quantitative Approach, Baltimore, The Johns Hopkins Press, 1967, pp. 131-148.

27 The "Varimax" method merely normalizes the results of the "Principle Axis" method. The loadings are the correlation coefficients between the index components and the factors.

TABLE IV

CORRELATION OF "FACTOR LOADED" QUALITY INDICES

	Index B
Index A (A)	-.05
" (B)	.03
" (C)	.017
" (D)	.0025
" (E)	.01

Note: Index A (A) is the new Index A arrived at by using the loadings in Factor A.

none of the weighting schemes make the Indices more comparable.

Given the lack of correlation between the quality indices the question naturally arises as to which is better suited for our purposes. Unfortunately, an unequivocal claim for supremacy cannot be made for either one from an intuitive or statistical point of view. Accordingly, both measures will be utilized in the study.

Summary

In this chapter we have briefly described the four basic methods of measuring educational quality. The two objective methods are 'product' and 'process' measures. The latter, in turn, can be approached from a 'quality of input' or 'adaptability' point of view. The fourth method is 'subjective' measurement. Because of the cost and difficulty involved in their construction, 'subjective' and 'adaptability' measures have not been used in this study.

A 'product' quality measure is constructed by using the results achieved by Ontario school children in achievement tests during their movement through the secondary school system in the years 1959-1964. The 'process' quality measure utilizes eight input variables and hypothesizes that if the quality of instructional inputs flowing into a system is high, then the quality of student output will also be high. Although these two indices have been constructed to measure quality of education they are not significantly correlated and an attempt to change this result by factor analytic methods has failed.

Since neither index can be said to be superior to the other they are both utilized in the following analysis and the effect of each on the cost curves is treated separately.

CHAPTER IV

ESTIMATES OF ECONOMIES OF SCALE

Introduction

Having analysed past studies and having developed two quality indices, a specific analysis of the Ontario Secondary School Education Sector can now be undertaken. This analysis has four aims. First, we discuss the general structure of the educational system in Ontario and extend our analysis to comment upon the shape which economic theory suggests the cost curves in this sector should have. Next, we describe the Public sub-sector and consider the quality of the data for this sub-sector and the procedures carried out to make it amenable to analysis. We then calculate specific regressions and analyse the results. Thirdly, the same procedure is carried out for the private and separate[1] sub-sectors and a comparison is made with the public sub-sector.

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1 The word 'separate' is used throughout this study as a reference to Catholic schools only.

Finally, an attempt is made to predict what effect the recent enlargement of school board district size in Ontario will have on costs.

The Structure of the Ontario Secondary School Education Sector

In 1964, the secondary school education sector in Ontario can best be described as containing three producing sub-sectors, the public, separate and private. The public school sub-sector is made up of 377 school boards, each of which operates zero,[2] one, or several schools. There are 299 separate and private schools. The separate schools offer grades 11-13. The boards which operate these schools receive no government subsidies and rely basically on tuition fees and parish grants for their operating funds.[3] The private schools are of three types: (a) independent schools, run mainly

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[2] Some boards act merely as accounting entities which pay tuition fees for students residing in their area of jurisdiction to other operating boards.

[3] Grades 9 and 10, in the separate school system are attached to catholic elementary school boards and received grants equal to those given to public elementary schools. These, in turn, are lower than those given to public secondary schools.

under church auspices; (b) business colleges; and (c) trade schools. Because of the data collection problem, only the independent schools listed in the Minister's Report[4] are treated in the study. The 131 trade schools and business colleges are omitted from consideration.

Given the structure of the secondary school sector, we now ask, what long-run cost relationships economic theory tells us to expect to find? In the case of school boards operating one school, and schools in the private and separate sub-sectors, we expect to find an 'L' shaped average cost curve. The vertical part of the 'L' owes its existence to small 'inefficient' schools which will, in turn, remain in use because of spatial demand factors. We do not expect to find a point where diseconomies come into being because at this point school boards will presumably erect an additional school.

A problem arises when we consider the case of school boards operating two or more schools. The question is whether there are economies of scale in

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4 Ontario Department of Education, Report of the Minister, 1964. Pp. S-160 - S-166.

multi-school school board operation. In particular, analysis such as that presented by Patinkin[5] suggests the same result as that presented in the single school case, i.e., even though schools can be represented as having 'adaptable'[6] fixed capital and thus 'U' shaped short run average cost curves, in the long run boards can just replecate their facilities. Thus, the long run, multi-school cost function will once again tend to be 'L' shaped.

However, certain forces might be present which can alter this simplistic result and lead to economies or diseconomies of scale. Among the possible forces are economies of large scale buying, technological economies and diseconomies of labour hiring. A source of the economy in buying might be the bulk purchase of texts or other instructional supplies. An example of a technological economy would be the installation of a completely integrated machine-teaching system. In the

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5 D. Patinkin, "Multiple Plant Firms, Cartels and Imperfect Competition," Quarterly Journal of Economics, February, 1947, pp. 173-205.

6 This concept is discussed in: G. Stigler, "Production and Distribution in the Short Run," Journal of Political Economy, June, 1939, pp. 305-327.

third case the emergence of a strong union as the size of the school board increases could be a latent source of diseconomy.

However, as noted by Hirsch,[7] we should expect no economies of large scale buying because the factors which the boards purchase are highly diversified and, in addition, they purchase almost none in quantity. He also notes that the second possible economy will also probably be unimportant because the importance of location tends to keep schools relatively small while legal restrictions on the level of salaries and permissable debt also tend to allow for only small technological economies.[8] With regard to manpower, size should not effect the relative teachers' salaries particularly in the public sub-sector because of the existence of the powerful Ontario Secondary School Teachers Federation. However, there might be a tendency for administrative personnel to unionize in larger schools thus leading to higher administrative costs. As a final factor,

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7 W. Z. Hirsch, "Analysis of the Rising Cost of Public Education," Joint Committee of the Congress of the United States, 89th Congress, 1st Session, Study Paper No. 4. U. S. Government Printing Office, 1959, p. 42.

8 Ibid., p. 42.

Hirsch notes that "large school districts . . . tend to lose efficiency because of political patronage and general administrative top-heaviness."[9]

Although the above analysis offers no firm theoretical conclusions as to the existence of economies of scale it does suggest that as a bare minimum we must look at possible economies in both school and school board operations. In addition, it will probably prove beneficial to make a further subdivision into the two functional subject areas, academic studies and commercial and vocational studies. This latter subdivision is advisable because the capital and labour requirements of each area are dissimilar enough so as to suggest unique educational processes.

The Public Sub-Sector[10]

The financial data for the public sub-sector have been gathered from balance sheets, income statements and

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[9] Ibid., p. 42.

[10] All analysis in this and the following sections relate to the school term 1963-1964.

attendance data filed with the Grants Section of the Ontario Department of Education by each secondary school board in the Province of Ontario and the types and numbers of boards in existence in the reporting period are summarized in Appendix D. It should be noted that the data have been obtained on the condition that they remain confidential and thus they are not reproduced as part of this study.

Because of certain inadequacies in the data, a number of operations are carried out before any analysis can be attempted. First, financial submissions of the 188 school boards offering both academic and commercial & vocational programmes have to be analysed. Each of these boards submit separate reports for their academic and commercial & vocational activities. This fact immediately gives rise to the possibility that costs attributable to academic and commercial & vocational activities might be divided only in the same ratio as the student ratio in these two activities.[11] If such a practice is in evidence it will make analysis of the

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[11] This division might be carried out for accounting convenience.

financial data by type of activity meaningless.

To ascertain the extent of this practice, the following test is carried out. First, for each of the 188 combined school boards a number of ratios are calculated. They are (a) Academic Average Daily Attendance (ADA)[12]/ Commercial & Vocational (C&V) ADA; (b)Academic administrative expense/ C&V administrative expense; (c) Academic plant maintenance expense/C&V plant maintenance expense; (d) Adademic instructor's salaries/C&V instructor's salaries; (e) Academic instructional supplies expense/C&V instructional supplies expense; (f) Academic total cost minus debt charges/ C&V total cost minus debt charges; and (g) academic total cost/C&V total cost. An analysis of the resulting ratios indicates that in only five cases are costs being divided in the same manner as the student ratios. Accordingly, the data submitted from the five boards following this practice are excluded from further consideration.

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12 Average Daily Attendance is defined as "pupil days attended divided by teaching days." This is the output measure recommended by Edding, F. Methods of Analysing Educational Outlay. UNESCO. Paris, 1966, p. 13. This is used as the raw output figure throughout the study.

The necessity for a second alteration arises from the fact that 32 of the 188 boards charge varying amounts of money to academic expense when it is in reality a transfer to the commercial & vocational revenue fund required to offset heavy commercial & vocational expenses. Accordingly, the requisite reductions are made in both the academic expense and commercial & vocational revenue funds in the data submitted from these boards.

The next requirement for an alteration arises from the fact that most of the 188 combined boards do not allocate their fixed capital separately to academic and commercial & vocational use. This division is necessary if the depreciation and tax charges mentioned in Chapter II are to be allocated to the appropriate cost functions. Fortunately, four schools do account for their fixed capital by educational use. Accordingly, calculations are made which indicate that the ratio of capital cost per commercial & vocational pupil to capital costs per arts pupil is 2.17.[13] This

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13 The ratios for the four boards are respectively 1.73, 1.97, 1.96, 3.02. Fortunately, the ratios are not correlated to school board size and thus no systematic bias enters into the estimate. In this case the smallest board does indeed possess the smallest ratio but the largest ratio is associated with the second smallest board.

ratio is used in all computations involving taxes and depreciation.[14]

A fourth alteration requires a number of school boards to be omitted from a detailed analysis because they are missing either (a) Carnegie data needed for the computation of a quality measure; or (b) capital cost measured at present value. To ascertain whether the ommissions might bias the results the following procedure is adopted.[15] First, a frequency distribution is constructed for all school boards operating either (a) both academic and commercial & vocational programmes; or (b) academic programmes only. Then, a second frequency distribution is constructed indicating the boards deleted for varying deficiencies. The resulting distributions are attached as Appendices

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[14] To ascertain the amount of capital associated with academic and commercial & vocational respectively, the following formulation is used, where:
X = Academic ADA Y = Commercial & Vocational ADA
A = Academic capital V = Commercial & Vocational Capital
K = Total Capital

$$A = \frac{X.K}{2.17Y + X} \qquad V = K - A$$

[15] The bias in this case would be the omission of an unduly large number of boards in any part of the size distribution.

E to I.[16] An examination of these appendices indicates that no bias appears to have arisen because of the deletions.

Finally, we should note the fact that in the analysis we use our quality indices in a multiplicative rather than in an additive manner. Specifically, we weight each school's (or school board's) Average Daily Attendance (ADA) figure by the appropriate index. By doing this the possibility is introduced that the distribution of school sizes which is provided by the raw data may differ somewhat from that provided by the quality-corrected data. If this is true then we could be faced with difficulties when attempting to apply our quality-corrected conclusions to the 'real world'. For example, in our quality-corrected data, economies of scale may be found in the range of 1000-1200 students. However, if the bulk of the observations

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[16] Note that in these Appendices a further subdivision has been made, i.e., of the 188 combined boards a division has been made between those operating one and more than one school.

are actually shifted into this range by quality corrections then how can we apply this finding to the 'real world'?

The best way to make the needed transition from quality-corrected ranges to raw data ranges would seem to be through the use of average quality, i.e., we would be standardizing for quality differences <u>ex post</u>. If we do this we will be making an 'other things being equal' type of assumption.

Having examined the difficulties presented by both the data and Indices A and B we are almost in a position to continue with our analysis. Before doing so, however, the form of the equations to be estimated must be specified. In particular, we use the following functional forms.[17]

$$TC = a + bX_1 + cX_1^2 + dX_1^3$$

$$TC = a + bX_2 + cX_2^2 + dX_2^3$$

$$TC = a + bX_3 + cX_3^2 + dX_3^3$$

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[17] Another interesting functional form is

$$TC = a + bX_1 + cX_1^2 + dX_1^3 + Q$$

where Q is a Quality Index. This equation and the associated topic of the "Cost-Quality Nexus" are discussed in Appendix J.

where TC = total cost as specified in Chapter II[18]

X_1 = Average Daily Attendance (ADA)

X_2 = ADA weighted by Index A

X_3 = ADA weighted by Index B

The three separate cost equations are being estimated so that a comparison can be made between the results produced by each.

Given these functional forms, a 'U' shaped cost curve results if parameter c is negative. An 'L' shaped curve will exist if $TC = a + bX$, i.e., if the squared and cubed variables can be omitted because of insignificance. In all cases our procedure is to first present the regression equation with all variables present regardless of their significance. Then, if applicable, an equation is presented which includes only significant variables.[19] Only equations

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18 Equations have also been estimated using Average Cost (AC) and the general form
$$AC = a/X + b + cX + dX^2.$$
The results of these fits are reported in Appendix K.

19 In this study it is considered that all results which fall within the 95% confidence level are significant. This follows the decision rule presented in: E. J. Kane, Economic Statistics and Econometrics. New York, Harper and Row, 1968, p. 210.

containing all significant variables are utilized in cost comparisons and diagmatic presentations. All equations are estimated using a stepwise regression program.

To systematize presentation of the results, each of the following four areas are analysed in turn: (a) All boards operating academic programmes; (b) All boards operating commercial & vocational programmes; (c) Boards operating academic programmes and one school only; (d) Boards operating commercial & vocational programmes and one school only.[20]

The estimation of the cost curves for all 161 boards operating academic programmes yields the following results.[21]

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[20] Analysis of (a) and (b) will yield an estimate of economies or diseconomies of scale in school board operation while an analysis of (c) and (d) will allow us to see if there are economies of scale in schools themselves.

[21] Throughout the study regressions are presented in the following manner:

Dependent Variable	=	Constant (Standard Error) [t Value]	+	Coefficients (Standard Error) [t Value]	(Independent variables)

Standard Error of Estimate (SE) R^2

$$\begin{array}{llllllll} TC = & -59530 & + & 790.1X_1 & - & .024X_1^{\;2} & + & .000001X_1^{\;3} \\ & (16940) & & (29.1) & & (.007) & & (.0000004) \\ & [-3.514] & & [27.14] & & [-3.40] & & [3.195] \end{array} \qquad (4\text{-}1)$$

$SE = 123{,}367$ $\qquad R^2 = .98944$

$$\begin{array}{llllllll} TC = & -46390 & + & 749.8X_2 & - & .01X_2^{\;2} & + & .0000002X_2^{\;3} \\ & (17800) & & (29.2) & & (.0064) & & (.0000003) \\ & [-2.61] & & [25.7] & & [-1.783] & & [0.72] \end{array} \qquad (4\text{-}2)$$

$SE = 133{,}224$ $\qquad R^2 = .98768$

$$\begin{array}{llllll} TC = & -38840 & + & 731.9X_2 & - & .007X_2^{\;2} \\ & (14360) & & (152.7) & & (.0013) \\ & [-2.7] & & [47.9] & & [-5.1] \end{array} \qquad (4\text{-}3)$$

$SE = 133{,}021$ $\qquad R^2 = .98764$

$$\begin{array}{llllllll} TC = & 65270 & + & 495.3X_3 & - & .0013X_3^{\;2} & - & .00000007X_3^{\;3} \\ & (16460) & & (21.6) & & (.0034) & & (.00000012) \\ & [3.97] & & [22.9] & & [-0.398] & & [-0.55] \end{array} \qquad (4\text{-}4)$$

$SE = 144{,}035$ $\qquad R^2 = .98560$

$$\begin{array}{llllll} TC = & 69100 & + & 487.2X_3 & - & .0000001X_3^{\;3} \\ & (13320) & & (7.5) & & (.000000025) \\ & [5.2] & & [64.95] & & [-4.6] \end{array} \qquad (4\text{-}5)$$

$SE = 143{,}651$ $\qquad R^2 = .98559$

The means of the costs for these boards are shown in Appendix L. The scatter diagrams of data and average cost curves for each of the relevant equations [(4-1), (4-3) and (4-5)] are exhibited in Appendices M, N and O.[22] The average cost equations which are plotted are the total cost equations divided by the relevant output measures. Table V shows the average cost figures produced by each of the relevant equations.[23]

The raw data and Index A-corrected data produce virtually the same shape of curve. It resembles a 'rotated J' (Γ). The only difference between the two arises in the fact that the peak of the curve occurs sooner in the raw data. The Index B-corrected data

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22 In all scatter diagrams the observations created by large variations in either quality index are deleted to allow a reasonable vertical scale to be constructed. The effect of the removal of these observations is reported in the text.

23 To make the costs comparable the raw output figures have been multiplied by the mean value of the relevant quality index before inserting these values into the cost equations produced by X_2 and X_3.

TABLE V

AVERAGE COST PRODUCED BY EACH OUTPUT MEASURE FOR 161 ACADEMIC BOARDS

Number of Students (Raw ADA)	$AC(X_1)$ $	$AC(X_2)$ $	$AC(X_3)$ $	Number of Students (Raw ADA)	$AC(X_1)$ $	$AC(X_2)$ $	$AC(X_3)$ $
200	488	538	833	6200	671	682	495
400	632	633	660	6400	667	681	494
600	677	664	602	6600	667	679	493
800	697	678	574	6800	665	678	493
1000	708	686	556	7000	663	677	492
1200	713	691	545	7200	661	676	492
1400	716	695	536	7400	660	674	491
1600	717	697	530	7600	658	673	491
1800	717	698	525	7800	657	672	490
2000	716	699	521	8000	655	671	489
2200	715	699	518	8200	654	669	489
2400	714	699	515	8400	653	668	488
2600	712	699	513	8600	651	667	488
2800	710	698	511	8800	650	665	487
3000	707	698	509	9000	649	664	487
3200	705	697	508	9200	648	663	486
3400	703	697	506	9400	647	661	486
3600	700	696	505	9600	647	660	485
3800	698	695	504	9800	646	659	485
4000	696	694	503	10,000	645	657	484
4200	693	693	502	11,000	643	651	481
4400	691	692	501	12,000	642	644	479
4600	688	691	500	13,000	644	637	476
4800	686	690	499	14,000	647	630	473
5000	684	689	499	15,000			469
5200	681	688	498	16,000			466
5400	679	687	497	17,000			462
5600	677	685	496	18,000			459
5800	675	684	496	19,000			455
6000	673	683	495	20,000			451

produces an average cost curve exhibiting continuing economies of scale. This result undoubtedly stems from the fact that raw size and Index B are positively and significantly correlated (r - .39).

The above results do not change significantly when so-called 'outlyer' output observations are taken into consideration. These 'outlyers' are observations which are created by extreme values of the quality indices. In our case 'outlyers' are considered to be ADA observations weighted by values of a quality index which are more than two standard deviations from the mean. After Index A outlyers are removed the relevant equations are:

$$\begin{array}{lllll} TC = -57240 & + \; 759.2X_2 & - \; .013X_2{}^2 & + \; .00000031X_2{}^3 & \quad (4\text{-}6) \\ \quad (17140) & (27.85) & (.006) & (.00000032) & \\ \quad [-3.34] & [27.3] & [-2.12] & [0.98] & \end{array}$$

SE = 126,119 $\qquad$ R^2 = .98915

$$\begin{array}{llll} TC = -47290 & + \; 736X_2 & - \; .007X_2{}^2 & \quad (4\text{-}7) \\ \quad (13800) & (14.5) & (.0013) & \\ \quad [-3.43] & [50.6] & [-5.6] & \end{array}$$

SE = 126,102 $\qquad$ R^2 = .98909

When Index B outlyers are removed the relevant equations are:

$$TC = 62940 + 504.7X_3 - .0027X_3^{\,2} - .00000002X_3^{\,3} \qquad (4\text{-}8)$$
$$\begin{array}{cccc} (16500) & (21.73) & (.0034) & (.00000012) \\ [3.8] & [23.2] & [-0.79] & [-0.19] \end{array}$$

$SE = 142,443$ $\qquad\qquad R^2 = .98624$

$$TC = 61310 + 508.3X_3 - .0034X_3^{\,2} \qquad (4\text{-}9)$$
$$\begin{array}{ccc} (14070) & (10.95) & (.00069) \\ [4.36] & [46.4] & [-4.86] \end{array}$$

$SE = 141,996$ $\qquad\qquad R^2 = .98624$

Examination of the equations indicates that the exclusion of outlyers does not change the basic shape of the total cost curves.

The general picture of slowly declining average costs which is produced by the total cost data is in most cases supported by the equations estimated from the average cost data which are reported in Appendix K. Although the raw data produces a 'rotated J' average cost curve when total cost data are used and constant costs when average cost data are used, only in the small size range are the curves in fact different. Over most of its range the curve produced by total costs is almost flat, i.e., it approximates constant average costs.

In the case of the average cost curve estimated using Index A-weighted average cost data we should not be too concerned that it does not support the general result of slowly falling average costs due to the high standard error generated by the equation.

It is interesting to note that when quality is used in an additive fashion in Appendix J, constant average costs are produced. This also gives some support to the general result of slowly declining average costs.

Moving now to a consideration of all 125 school boards in the commercial & vocational area we find the following results.

$$\begin{array}{lllll} TC = & 44980 & + \; 496.9X_1 & + \; .31X_1^{\;2} & - \; .00004X_1^{\;3} \\ & (25740) & (68.3) & (.032) & (.0000035) \\ & [1.75] & [7.3] & [9.7] & [-11.2] \end{array} \qquad (4\text{-}10)$$

$SE = 170,609$ $\qquad$ $R^2 = .97639$

$$TC = \underset{\substack{(24380)\\ [1.88]}}{45940} + \underset{\substack{(64.3)\\ [7.66]}}{492.3X_2} + \underset{\substack{(.03)\\ [9.78]}}{.295X_2^{\,2}} - \underset{\substack{(.0000033)\\ [-11.2]}}{.000037X_2^{\,3}} \qquad (4\text{-}11)$$

$$SE = 160{,}455 \qquad\qquad R^2 = .97912$$

$$TC = \underset{\substack{(30890)\\ [2.6]}}{80610} + \underset{\substack{(65.1)\\ [6.8]}}{443.2X_3} + \underset{\substack{(.022)\\ [4.9]}}{.11X_3^{\,2}} - \underset{\substack{(.0000018)\\ [-5.7]}}{.00001X_3^{\,3}} \qquad (4\text{-}12)$$

$$SE = 228{,}796 \qquad\qquad R^2 = .95755$$

The means of the costs for these boards are exhibited in Appendix P, while the scatter diagrams and average cost curves for each of the equations are shown in Appendices Q, R and S. The average costs produced by each equation are shown at Table VI.

In all three cases a 'Lazy S' ( ) average cost curve is produced. The only difference in shape between the three is the size range at which the highest cost occurs. The raw data and Index A-weighted curves peak at 3800 students while the Index B-weighted curve peaks at 5400 students. The fact that this latter curve peaks at all is surprising because of the previously mentioned significant correlation between Index B and ADA.

TABLE VI

AVERAGE COST PRODUCED BY EACH OUTPUT
MEASURE FOR 125 COMMERCIAL & VOCATIONAL BOARDS

Number of Students (Raw ADA)	$AC(X_1)$ $	$AC(X_2)$ $	$AC(X_3)$ $	Number of Students (Raw ADA)	$AC(X_1)$ $	$AC(X_2)$ $	$AC(X_3)$ $
200	782	776	852	5200	1036	1024	760
400	727	719	680	5400	1013	1002	760
600	743	734	637	5600	987	977	758
800	776	765	625	5800	957	949	756
1000	812	800	624	6000	924	918	753
1200	849	836	630	6200	889	884	749
1400	885	871	638	6400	850	847	744
1600	919	904	647	6600	807	806	738
1800	950	935	657	6800	762	763	732
2000	979	963	668	7000	713	717	724
2200	1006	989	678	7200	662	667	716
2400	1029	1012	688	7400	607	615	707
2600	1050	1032	698	7600	548	559	697
2800	1067	1049	707	7800			674
3000	1082	1064	715	8000			662
3200	1093	1075	723	8200			648
3400	1102	1084	730	8400			634
3600	1107	1089	737	8600			619
3800	1109	1092	743	8800			603
4000	1108	1091	747	9000			586
4200	1104	1088	752	9200			568
4400	1097	1081	755	9400			549
4600	1086	1071	758	9600			530
4800	1073	1059	759	9800			510
5000	1056	1043	760	10,000			488

As in the case of the 161 academic boards the exclusion of 'outlyers' does not change the basic results. In this case the equations produced when outlyers are omitted are:

$$TC = \underset{\substack{(24040)\\[1.8]}}{43310} + \underset{\substack{(63.0)\\[7.7]}}{487.7X_2} + \underset{\substack{(.0295)\\[10.1]}}{.298X_2^{\,2}} - \underset{\substack{(.0000033)\\[-11.5]}}{.000038X_2^{\,3}} \qquad (4\text{-}13)$$

$SE = 156{,}279$ $\qquad R^2 = .98047$

$$TC = \underset{\substack{(29410)\\[2.09]}}{61640} + \underset{\substack{(63.2)\\[8.1]}}{512.6X_3} + \underset{\substack{(.021)\\[4.5]}}{.097X_3^{\,2}} - \underset{\substack{(.0000018)\\[-5.59]}}{.0000099X_3^{\,3}} \qquad (4\text{-}14)$$

$SE = 214{,}372$ $\qquad R^2 = .96383$

The average cost data also produce 'Lazy S' average cost curves. This is again the case when quality is used additively. These results are reported in Appendices J and K.

When regressions are run on the data provided by the 113 academic boards operating only one school, the following equations are produced:

$$TC = \underset{\substack{(36330)\\[1.1]}}{40360} + \underset{\substack{(322.3)\\[0.79]}}{254.4X_1} + \underset{\substack{(.86)\\[1.2]}}{1.04X_1^{\,2}} - \underset{\substack{(.00071)\\[-1.2]}}{.00088X_1^{\,3}} \qquad (4\text{-}15)$$

$SE = 36{,}602$ $\qquad R^2 = .86864$

$$TC = 627.1X_1 \quad (4\text{-}16)$$
$$\begin{array}{c} (8.96) \\ [70.01] \end{array}$$

SE = 36,410 $\qquad R^2 = .86644$

$$TC = 73430 + 3.4X_2 + 1.54X_2^{\,2} - .0012X_2^{\,3} \quad (4\text{-}17)$$
$$\begin{array}{cccc} (25680) & (223.1) & (.595) & (.00049) \\ [2.86] & [0.015] & [2.59] & [-2.48] \end{array}$$

SE = 37,599 $\qquad R^2 = .86139$

$$TC = 612X_2 \quad (4\text{-}18)$$
$$\begin{array}{c} (9.29) \\ [65.9] \end{array}$$

SE = 38,637 $\qquad R^2 = .84960$

$$TC = 91370 + 231.8X_3 + .847X_3^{\,2} - .0009X_3^{\,3} \quad (4\text{-}19)$$
$$\begin{array}{cccc} (22810) & (212.1) & (.54) & (.0004) \\ [4.01] & [1.09] & [1.56] & [-2.25] \end{array}$$

SE = 57,202 $\qquad R^2 = .67917$

$$TC = 62580 + 552.2X_3 - .00029X_3^{\,3} \quad (4\text{-}20)$$
$$\begin{array}{ccc} (13540) & (54.6) & (.000085) \\ [4.62] & [10.12] & [-3.43] \end{array}$$

SE = 57,576 $\qquad R^2 = .67199$

The scatters of data and plots of the average cost curves for these equations are shown at Appendices U, V and W. The means of the costs are exhibited at Appendix T. The costs produced by equation (4-20) are tabulated at Table VII. Since the other relevant equations [(4-16) and (4-18)] produce constant average costs their results are not included in this Table.

For these 113 boards, the raw data and Index A-weighted data produce constant costs while the Index B-weighted data produce a continually declining average cost curve or economies of scale. The latter result is aided by the significant correlation between ADA and Index B.

Again, outlyers have little effect on the results. The equations produced when the outlyers are removed are as follows:

$$\underset{\substack{(4436)\\[0.08]}}{TC = 362.1} + \underset{\substack{(78.9)\\[8.38]}}{661.1X_2} - \underset{\substack{(.338)\\[-0.53]}}{.18X_2^{\,2}} + \underset{\substack{(.00034)\\[0.27]}}{.000089X_2^{\,3}} \qquad (4\text{-}21)$$

$SE = 28{,}410$ $\qquad R^2 = .94053$

TABLE VII

AVERAGE COST PRODUCED BY INDEX
B-WEIGHTED OUTPUT FOR 113 ACADEMIC BOARDS

Number of Students (Raw ADA)	AC $
100	1251
200	893
300	765
400	690
500	635
600	586
700	539
800	492

$$TC = 601.9X_2 \qquad (4\text{-}22)$$
$$\begin{array}{l} (10.1) \\ [59.8] \end{array}$$

$SE = 28{,}505$ $\qquad\qquad R^2 = .93847$

$$TC = 180000 \;-\; .018X_3 \;+\; .000000005X_3^{\,2}$$
$$\begin{array}{ccc} (8988) & (.004) & (.0000000002) \\ [20.03] & [-4.51] & [2.55] \end{array} \qquad (4\text{-}23)$$
$$-\; .00000000000000005X_3^{\,3}$$
$$\begin{array}{c} (.000000000000000026) \\ [-1.85] \end{array}$$

$SE = 75{,}241$ $\qquad\qquad R^2 = .57858$

As reported in Appendix K the same types of average cost curves are produced when average cost data are used except in the case of Index A-weighted data. Here the average cost data produce constantly falling average costs, a fact which is in conflict with the constant average costs produced by the total cost data. It should be noted, however, that the coefficients of variation of the equations produced by average cost and total cost data respectively are .22 and .17. Thus, we might give a little more credence to the equation produced by the total cost data. In any case, both sets of data produce average cost curves which do not negate our *a priori* premise of the absence of diseconomies of large scale in schools.

The intriguing aspect of the cost curves for single-school operation is that they tend to give somewhat lower costs than the cost curves for all boards over the range in which there are single school operations. To approximate the extent of this difference, the average cost equations for the 161 academic boards are estimated using dummy variables to indicate whether a school board operates one or more than one school. The value of the coefficient for the dummy variable will indicate the change in average cost per pupil which is in evidence because of the existence of multiple school boards. The following functional forms are used:

$$AC_1 = a + \frac{b}{X_1} + cX_1 + dX_1^{\,2} + eD$$

$$AC_2 = a + \frac{b}{X_2} + cX_2 + dX_2^{\,2} + eD$$

$$AC_3 = a + \frac{b}{X_3} + cX_3 + dX_3^{\,2} + eD$$

where $AC_1 = \frac{TC}{X_1}$

$AC_2 = \frac{TC}{X_2}$

$AC_3 = \frac{TC}{X_3}$

D = 1 if a board operates more than one school.

D = 0 if a board operates one school only.

Average cost curves are estimated because in a total cost formulation the effect of the dummies tends to be overshadowed by the large values of the other variables. This same problem is discussed in Appendix J.

The pertinant estimations are as follows:

$$AC_1 = \frac{10385}{X_1} + 583.5 + .039X_1 - .000003X_1^{\,2} + 27.43D \tag{4-24}$$

$$\begin{array}{ccccc} (6697) & (30.15) & (.024) & (.816) & (.0000021) \\ {[1.55]} & {[19.36]} & {[1.61]} & {[0.82]} & {[-1.40]} \end{array}$$

$SE = 129.8$ $\qquad\qquad R^2 = 0.04439$

$$AC_1 = 633.3 + 41.5D \tag{4-25}$$

$$\begin{array}{cc} (12.24) & (22.4) \\ {[51.73]} & {[1.85]} \end{array}$$

$SE = 130.13$ $\qquad\qquad R^2 = .02110$

$$AC_2 = \frac{121000}{X_2} + 165.1 + .11X_2 - .0000077X_2^{\,2} + 267.7D \tag{4-26}$$

$$\begin{array}{ccccc} (425.5) & (25.94) & (.036) & (.0000029) & (57.08) \\ {[284.3]} & {[6.36]} & {[3.01]} & {[-2.64]} & {[4.69]} \end{array}$$

$SE = 236.7$ $\qquad\qquad R^2 = 0.99808$

$$AC_3 = \frac{104200}{X_3} + 319.6 + .0047X_3 - .0000004X_3^{\,2} + 162.7D \tag{4-27}$$

$$\begin{array}{ccccc} (5019) & (42.08) & (.031) & (.0000018) & (72.98) \\ {[20.77]} & {[7.59]} & {[0.15]} & {[-0.22]} & {[2.23]} \end{array}$$

$SE = 305.4$ $\qquad\qquad R^2 = 0.75560$

$$AC_3 = \frac{104100}{X_3} + 321.8 + 165.6D \tag{4-28}$$

$$\begin{array}{ccc} (4885) & (38.89) & (56.77) \\ {[21.31]} & {[8.27]} & {[2.92]} \end{array}$$

$SE = 303.5$ $\qquad\qquad R^2 = .75548$

Thus, regardless of which output measure is used, the positive and significant values of the dummy variables indicates that the existence of multiple school boards does raise the average cost of secondary school education.

Turning now to our last area of concern in the public sub-sector we find that the data produced by the 77 school boards operating a commercial & vocational programme and only one school produce the following equations:

$$\begin{aligned} TC = \underset{\substack{(7617)\\ [0.104]}}{793.2} + \underset{\substack{(91.6)\\ [7.76]}}{710.5X_1} + \underset{\substack{(.26)\\ [1.76]}}{46X_1^{\,2}} - \underset{\substack{(.00019)\\ [-2.99]}}{.00058X_1^{\,3}} \end{aligned} \qquad (4\text{-}29)$$

$$SE = 25{,}236 \qquad \bar{R}^2 = .97246$$

$$\begin{aligned} TC = \underset{\substack{(5283)\\ [-1.704]}}{-9000} + \underset{\substack{(24.7)\\ [35.03]}}{866.2X_1} - \underset{\substack{(.000038)\\ [-6.574]}}{.00025X_1^{\,3}} \end{aligned} \qquad (4\text{-}30)$$

$$SE = 25{,}592 \qquad \bar{R}^2 = .97129$$

$$TC = -1582 + 738.3X_2 + .267X_2^2 - .004X_2^3 \qquad (4\text{-}31)$$
$$\begin{array}{cccc} (8573) & (103.5) & (.299) & (.00022) \\ [-0.18] & [7.13] & [0.89] & [-1.83] \end{array}$$

SE = 27,954 $\qquad$ $R^2 = .96621$

$$TC = 800.5X_2 - .00019X_2^3 \qquad (4\text{-}32)$$
$$\begin{array}{cc} (15.9) & (.000034) \\ [50.31] & [-5.47] \end{array}$$

SE = 28,012 $\qquad$ $R^2 = .96514$

$$TC = 22530 + 730.9X_3 + .08X_3^2 - .00025X_3^3 \qquad (4\text{-}33)$$
$$\begin{array}{cccc} (12750) & (148.0) & (.38) & (.000248) \\ [1.77] & [4.94] & [0.21] & [-1.01] \end{array}$$

SE = 50,010 $\qquad$ $R^2 = .89186$

$$TC = 20700 + 760.5X_3 - .0002X_3^3 \qquad (4\text{-}34)$$
$$\begin{array}{ccc} (9202) & (41.4) & (.000049) \\ [2.25] & [18.36] & [-4.10] \end{array}$$

SE = 49,686 $\qquad$ $R^2 = .89179$

The means of the costs for these boards are shown at Appendix X while the scatters of data and relevant average cost curves are exhibited in Appendices Y, Z and AA. The costs estimated by each of the equations are tabulated in Table VIII.

TABLE VIII

AVERAGE COST PRODUCED BY EACH OUTPUT
MEASURE FOR 77 COMMERCIAL & VOCATIONAL BOARDS

Number of Students (Raw ADA)	AC (X_1) $	AC (X_2) $	AC (X_3) $
100	774	798	985
200	813	792	867
300	818	782	821
400	812	768	790
500	798	750	764
600	779	728	738
700	755	701	711
800	727	671	681
900	694	636	650
1000	657	598	615

In this area, the raw data produces a cost curve shaped like an inverted U. The peak in cost occurs at about 300 students. On the other hand, both cost curves obtained using the Index-weighted data exhibit continuing economies of scale.

There are no Index A 'outlyers' in this data. The deletion of Index B 'outlyers' produces the following equations:

$$\begin{array}{lllll} TC = 10070 & + \; 828.9X_3 & - \; .12X_3^2 & - \; .00014X_3^3 & \\ \quad (10530) & \quad (121.1) & \quad (.31) & \quad (.0002) & (4\text{-}35) \\ \quad [0.96] & \quad [6.84] & \quad [-0.37] & \quad [-0.71] & \end{array}$$

$SE = 40,569 \qquad R^2 = .92927$

$$\begin{array}{llll} TC = 12780 & + \; 785.6X_3 & - \; .00022X_3^3 & \\ \quad (7575) & \quad (33.9) & \quad (.00004) & (4\text{-}36) \\ \quad [1.69] & \quad [23.2] & \quad [-5.47] & \end{array}$$

$SE = 40,329 \qquad R^2 = .92914$

Equation (4-36) produces results very similar to (4-34). The resulting average cost curve is still downward sloping.

Appendix K indicates that when average cost data is used the shape of the average cost curve which is estimated differs from the shape of that produced by the total cost data in two cases. In the case of Index A-weighted data, average cost data produce an average cost curve which first rises and then falls. The average cost curve produced by total cost falls continually. Over most of the relevant size range however, both average cost curves move in the same direction, downward.

When raw data is utilized falling average costs are produced by the total cost data whereas the average cost data produce a flat average cost curve. Neither average cost curve seems to be clearly superior to the other statistically. However, they both support our *a priori* premise that we will not find increasing costs in the single school case.

As was done in the case of the 161 academic school boards, the equations for the 125 commercial &

vocational boards are reestimated using dummy variables to see if boards operating more than one school do have higher costs. The results are:

$$AC_1 = \frac{2027}{X_1} + 729.3 + .13X_1 - .00002X_1^2 + 29.54D \qquad (4\text{-}37)$$

$$\begin{matrix} (999.9) & (24.62) & (.044) & (.000007) & (36.97) \\ [2.03] & [29.63] & [2.95] & [-2.55] & [.79] \end{matrix}$$

SE = 146.73 $\qquad$ $R^2 = .15417$

$$AC_1 = 782.9 + 91.03D \qquad (4\text{-}38)$$

$$\begin{matrix} (17.22) & (27.79) \\ [45.46] & [3.28] \end{matrix}$$

SE = 151.1 $\qquad$ $R^2 = .08021$

$$AC_2 = \frac{6027}{X_2} + 666.3 + .09X_2 - .000013X_2^2 + 136.1D \qquad (4\text{-}39)$$

$$\begin{matrix} (978) & (29.44) & (.056) & (.0000086) & (46.92) \\ [6.16] & [22.63] & [1.66] & [-1.51] & [2.90] \end{matrix}$$

SE = 186.7 $\qquad$ $R^2 = .31956$

$$AC_2 = \frac{5418}{X_2} + 694 + 189.6D \qquad (4\text{-}40)$$

$$\begin{matrix} (910.7) & (24.05) & (35.15) \\ [5.95] & [28.90] & [5.32] \end{matrix}$$

SE = 187.4 $\qquad$ $R^2 = .30365$

$$AC_3 = \frac{9668}{X_3} + 940.2 - .22X_3 + .000021X_3^2 + 36.83D \qquad (4\text{-}41)$$

$$\begin{array}{ccccc} (2899) & (124.1) & (.189) & (.000022) & (224) \\ [3.33] & [7.58] & [-1.14] & [.99] & [.16] \end{array}$$

$SE = 880.4$ $\qquad R^2 = .13209$

Only in the last case do we find the effect of multiple school boards on average cost to be insignificant. Even in this case, however, it still has a positive sign. In summary, then, the results in this area are the same as those found when considering academic school boards. Multiple school boards generate a higher cost per student.

The Private and Separate Sub-Sector

As relevant figures for this sub-sector are not available publicly, primary data collection had to be undertaken. This task has been carried out in the following manner. First, questionnaires were constructed for each of the two types of schools (independent and separate). These questionnaires were checked with educators employed in these areas in the London, Ontario area. The resulting forms, amended according to suggestions of the educators, are attached as Appendices BB and CC. Appendix BB was mailed to separate schools and Appendix CC to independent schools. Attached as Appendix DD is the author's covering letter and as Appendix EE, a letter from the author's thesis supervisor.

Questionnaires were mailed to all independent and separate schools listed in the Minister's Report of 1964,[24] and which were still in existence in 1968. In total, 137 questionnaires were mailed. After approximately eight weeks, a follow-up letter was mailed to schools which had not yet replied. A copy of this letter is attached as Appendix FF. As of the writing of this thesis the following is the 'return state' of the questionnaires.

Number not returned		68
Returned but not completed[25]		47
Number of completed returns:		
Private schools	- 10	
Separate schools	- 12	22
		137

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24 Ontario Department of Education, Report of the Minister, 1964, op. cit., pp. S-160 - S-166.

25 The majority of the respondees quote a poor accounting system or lost data as the reason for non-completion.

Although this 16% completion rate is by no means completely satisfactory an attempt has nevertheless been made to analyse the data from these schools.

To put the received data into useable form, two operations must be carried out. First, quality indices have to be constructed. Since pupils in 19 of the respondees' schools have written the Carnegie tests, no problem exists here. The results from these independent and separate schools are pooled with those from the public schools and Index A is computed for each of the 19 schools. Unfortunately, a number of the returned questionnaires are lacking data necessary to compute Index B so this index is not constructed.

Second, for a number of the separate schools, imputations for teachers' salaries have to be carried out. This is necessary because a number of the teachers are members of a religious order and work without salary. In the eight schools effected, instructional salaries are estimated in the following manner. Help wanted advertisements for teachers for Public Secondary Schools in the relevant localities are consulted. In particular, the advertisements which appear in the 1964 February 24th issue of the Toronto Globe and Mail are used as the data source. The 'level one' rate of salary is used as the basis and years experience are allowed for up to the five year maximum. This method tends to give a downward bias to the estimate of salaries because specialist degrees and higher educational specialization are not allowed for. However, it does serve as an approximation.

Unfortunately, five of the remaining 19 schools have to be dropped from the analysis because no capital figures are available. The regression results from the remaining 14 schools are as follows:

$$\begin{array}{lllll} TC = & 242800 & - \; 2697X_1 & + \; 16.26X_1^2 & - \; .018X_1^3 \\ & (315900) & (35890) & (11.24) & (.01) \\ & [.77] & [-.75] & [1.45] & [-1.8] \end{array} \qquad (4\text{-}42)$$

$SE = 170{,}587$ $\qquad R^2 = .61634$

$$\begin{array}{lll} TC = & 1518X_1 & - \; .002X_1^3 \\ & (320.9) & (.001) \\ & [4.73] & [-2.0] \end{array} \qquad (4\text{-}43)$$

$SE = 182{,}615$ $\qquad R^2 = .47240$

$$TC = 92200 - 1023X_2 + 11.1X_2^2 - .014X_2^3 \qquad (4\text{-}44)$$

$$\begin{array}{cccc} (15810) & (1865) & (6.58) & (.007) \\ [.58] & [-.55] & [1.69] & [-2.00] \end{array}$$

$SE = 171,153$ $\qquad R^2 = .61379$

$$TC = 14960X_2 - .002X_2^3 \qquad (4\text{-}45)$$

$$\begin{array}{cc} (321.8) & (.001) \\ [4.65] & [-2.00] \end{array}$$

$SE = 181294$ $\qquad R^2 = .48000$

The means of the costs for this sector are attached as Appendix GG, the summary of costs per student are shown at Table IX and the scatters and plots are attached as Appendices HH and II. The scatter contains only 13 points because of an 'outlyer'. One school has a quality index of .008. If a regression is run minus this observation, the significant results are:

$$TC = 333200 - 3469X_2 + 17.9X_2^2 - .019X_2^3 \qquad (4\text{-}46)$$

$$\begin{array}{cccc} (414900) & (4322) & (12.7) & (.011) \\ [.80] & [-.80] & [1.41] & [-1.75] \end{array}$$

$SE = 176,538$ $\qquad R^2 = .60041$

$$TC = 1496X_2 - .002X_2^3 \qquad (4\text{-}47)$$

$$\begin{array}{cc} (335.2) & (.001) \\ [4.46] & [-2.0] \end{array}$$

$SE = 188,821$ $\qquad R^2 = .44128$

TABLE IX

AVERAGE COSTS PRODUCED BY OUTPUT MEASURES FOR 14 SCHOOLS IN THE PRIVATE AND SEPARATE SUB-SECTORS

Number of Students (Raw ADA)	AC (X_1) $	AC (X_2) $
100	1498	1475
200	1438	1414
300	1338	1311
400	1198	1167
500	1018	982
600	798	756
700	538	488

Equations (4-43) and (4-45) produce continually declining costs.

Appendix K indicates the average cost data produce the same result in the case of the Index A-weighted figures. However, the raw average costs produce a constant average cost curve. Neither of the conflicting equations are clearly superior statistically but they both do support our *a priori* premise of no diseconomies of large scale in schools.

The important question in this case is whether the cost equations produce average cost curves which are significantly different from those produced in the public sub-sector, i.e., is the private and separate sub-sector more efficient than the public sub-sector? To analyse this question the data from the 14 schools in this section are pooled with that produced by the 35 schools in the academic area which offer only an academic programme.[26] In this case the dummy variable is given a value of one if a school is in the private and separate sub-sector and a value of zero means a school is an 'academic only'. The pooled data produce the following equations:

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[26] These schools are used since their operations most closely approximate those of the independent and separate schools.

$$AC_1 = -\frac{23540}{X_1} + 736.9 + .6X_1 - .002X_1^2 + 517D \qquad (4\text{-}48)$$

$$\begin{array}{ccccc} (55420) & (832.4) & (3.52) & (.004) & (127.3) \\ [-.42] & [.88] & [.17] & [-.50] & [4.06] \end{array}$$

SE = 363 $\qquad R^2 = .27732$

$$AC_1 = \underset{\substack{(61.3)\\ [10.10]}}{619.1} + \underset{\substack{(114.7)\\ [3.78]}}{433.7D} \qquad (4\text{-}49)$$

SE = 363 $\qquad R^2 = .23327$

$$AC_2 = \frac{47170}{X_2} + 12020 - 73.95X_2 + .11X_2^2 - 2668D \qquad (4\text{-}50)$$

$$\begin{array}{ccccc} (3079) & (3424) & (23.26) & (.036) & (1854) \\ [15.32] & [3.512] & [-3.18] & [2.95] & [-1.44] \end{array}$$

SE = 533 $\qquad R^2 = .91043$

This result changes markedly when two 'outlyers' are removed from the sample. When this is done the Index A-weighted data produce these equations:

$$AC_2 = \frac{20530}{X_2} + 195.2 + 2.43X_2 - .004X_2^2 + 510.9D \qquad (4\text{-}51)$$

$$\begin{array}{ccccc} (54430) & (825) & (3.50) & (.004) & (129.0) \\ [.38] & [.24] & [.69] & [1.00] & [3.96] \end{array}$$

SE = 357 $\qquad R^2 = .27653$

$$AC_2 = \underset{\substack{(60.7) \\ [10.23]}}{620.7} + \underset{\substack{(115.3) \\ [3.76]}}{433.1D} \qquad (4\text{-}52)$$

$SE = 354$ $\qquad\qquad R^2 = .23864$

The positive value of D indicates that schools in the private and separate sub-sector have a higher average cost than similar schools in the public sub-sector. Unfortunately, the small sample size and existence of imputations in the private and separate sub-sectors detract from the significance of this result.

Scale Effects of School Board Reorganization in Ontario

Since our results indicate that there are areas of diseconomies of scale in school board operation in all but one case (when X_3 output is used in the analysis of all academic boards) the question of the effect on average costs of the increase in school board size, caused by school board reorganization in Ontario, naturally arises. From a simplistic point of view, our analysis, in general, suggests that only if school boards reach a size of approximately 6000-7000 students in each of the academic and commercial &

vocational programmes will average cost per student be lower than in any other size class. The 1967 report of the Minister of Education[27] indicates that in only 8 of 50 cases do school districts have a total secondary enrolment of more than 12,000 students. Thus there would appear to be no clear cut argument for consolidation of boards from an economies of scale point of view.

In an effort to obtain a more specific estimate of the scale effects of consolidation the following analysis is carried out. We look at the new (1969) enlarged school board districts and ascertain what smaller school boards existed within these districts in 1964. The actual costs of these 1964 school boards are totalled for each new school district and compared to the costs predicted by our regression equations for a school board of the size given by the total attendance figures (the summation of attendance figures for smaller boards which are now in the area of jurisdiction of the larger boards). We can then compare the two

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[27] Ontario Department of Education, Report of the Minister of Education, 1967, 1968, p. 55.

cost figures. Unfortunately, we only have complete data for eight of the new enlarged school boards.[28] This sample can be increased by three by the addition of enlarged school districts for which only the data for one 1964 school board is missing. This omitted board accounts for less than 5% of total secondary school attendance in the new board's jurisdiction.

Table X gives the size distribution of all boards and of the eleven boards which make up our sample. It indicates that our sample has a lack of representation of larger boards and to the extent that it is among these that our estimates predict economies then our results might be somewhat biased toward overstating diseconomies.

The cost ratios are summarized in Table XI. They indicate that output measures X_1 and X_2 predict substantial increases in costs with consolidation. Measure X_3, undoubtedly because of its significant correlation with size, indicates substantial economies.

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[28] The problems mentioned above, e.g., lack of Carnegie data, lack of capital figures, are the cause of this limitation.

TABLE X

SIZE DISTRIBUTION OF SCHOOL BOARDS IN 1964
BASED ON 1969 BOUNDARIES[1]

ADA	Academic[2]	Commercial & Vocational
0-1000	6	15 (3)
1001-2000	9 (5)	12 (4)
2001-3000	9 (1)	14 (2)
3001-4000	14 (1)	4 (2)
4001-5000	0 (3)	3
5001-6000	7 (1)	1
6001-7000	5	0
7001-8000	1	2
8001-9000	0	0
9001-10,000	0	0
10,001-11,000	0	0
11,001-12,000	0	0
Over 12,000	3	2

[1] Bracketed numbers are the frequencies occuring in the sample of 11 boards used to calculate the percentages used in Table XI.

[2] This distribution has one more value than that for commercial & vocational because one of the boards had academic students only.

TABLE XI

PREDICTED COSTS AS A PERCENTAGE OF ACTUAL COSTS

Academic Operations

	Output Measure		
	X_1	X_2	X_3
Average - Eight Complete Boards	19.1	6.1	-17.8
Average - Eleven Boards	13.5	3.0	-13.5

Commercial & Vocational Operations

	Output Measure		
	X_1	X_2	X_3
Average - Eight Complete Boards	14.5	9.3	-10.5
Average - Eleven Boards	11.6	8.6	- 9.

Summary

In this chapter we have examined the shape of the cost functions in different areas of the secondary school education sector in Ontario. In the case of single schools we find, with one exception, either continuing economies of scale (falling average costs) or constant returns to scale (constant average costs). This result backs up our <u>a priori</u> theoretical reasoning that we should not expect to find areas of decreasing returns (increasing average costs) in single school operation. However, when the possibility of the existence of economies of scale in school boards is investigated, the results are not so clear cut. When output is measured by raw attendance data or the raw data weighted by a 'product' quality index we find 'rotated J' ( ) shaped average cost curves for the academic boards and 'Lazy S' ( ) shaped average cost curves for the commercial & vocational boards. When raw attendance data is weighted by an 'input' quality measure the result is increasing returns to scale in the academic case and a 'Lazy S' average cost curve in the commercial &

vocational case. The existence of the diseconomies portions of both the 'Lazy S' and the 'Rotated J' curves have no theoretical explanation.[29]

The use of dummy variables with the cost data has been responsible for two interesting results. First, we find that cost per student is higher for school boards operating more than one school than it is for school boards operating one school only. Second, it appears that the cost of educating students in independent and separate schools is greater than the cost in public schools. In the latter case however, our small sample size and relatively poor fits for the independent and separate schools detract from the confidence which we can place in this result.

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[29] It has been suggested to the author that they might be caused by some unadaptable factors of production such as administrative specialists.

CHAPTER V

SUMMARY AND CONCLUSIONS

An attempt has been made in this study to ascertain whether or not economies of scale do exist in the secondary school education sector in the Province of Ontario. Cost regressions have been estimated using both Average Daily Attendance (ADA) and ADA weighted by quality indices as measures of output. We have found that single schools generally have either constant average costs or continually declining average costs. Thus, either constant returns to scale or increasing returns to scale are present in this case.

When school boards are examined we find, in general, areas of both economies and diseconomies of scale in both the academic and commercial & vocational areas of study. In only one case are continuing economies of scale present, that is when ADA weighted by an 'input' quality measure is used as output in the academic area of studies. The results indicate that only when a school board reaches a size of approximately 6000-7000 students in both academic and commercial & vocational programmes are costs likely to be lower than in any other size range. It must be noted, however, that this finding is produced by a sample in which there are only a very few school boards in this size range.

An effort has been made to ascertain the scale cost effects of the Ontario Government's recent reorganization of school boards into larger areas of jurisdiction. Our raw data and data weighted by a 'product' quality index indicate that costs will increase by from 3% to 19% depending on the area of education. On the other hand when ADA weighted by an 'input' quality measure is used as the measure of output cost reductions of from 9% to 18% are predicted.

Two interesting results have been produced by the use of dummy variables. First, school boards operating one school only appear to have substantially lower costs per pupil than school boards operating more than one school. Second, separate and independent schools have higher costs per pupil than schools which carry out a similar function (academic studies) in the public sector. However, the small sample size and existence of imputations in the independent and separate data force us to state this conclusion somewhat tentatively.

It should be pointed out that some of the cost data have been based on a rather arbitrary division of the capital stock within school boards. Another division might cause some change in the final results.

The primary need for further research would appear to be in the 'post consolidation' cost structure in the public secondary sector. Since school board districts will be much larger, one will be able to obtain more observations in the over 12,000 student size range and thus it will be possible to see with a greater degree of accuracy whether economies of scale do, in fact, exist in this area.

APPENDIX A

SUMMARY OF "PROCESS" QUALITY MEASURES

Measure	Source	Comment
1. Length of school year.	H. F. Clark, Research in Educational Finance. "Utilization of Educational Resources: Cost-Quality Relationships in Public Education." Syracuse, N. Y., Syracuse University Press, p. 3.	This is a measure used in time series given that schools are operated under common legislation.
	W. E. Barron, "Measurement of Education-Productivity," in: The Theory and Practice of School Finance, W. E. Gauerke and J. R. Childress (Eds.). Chicago, Rand-McNally, 1967, p. 291.	
	J. A. Thomas, "Efficiency in Education: An Empirical Study," in: Administrator's Notebook. Chicago, Midwest Administrative Centre, 1962.	
2. Length of school day.	Clark, op. cit., p. 50.	As above.

3. Class size.	M. Blaug and M. Woodhall, "Productivity Trends in British University Education, 1938-1962," Minerva, Vol. 3, 1962, p. 497. J. Burkhead, Input and Output in Large City High Schools. Syracuse, N. Y., Syracuse University Press, 1967, p. 32. W. Z. Hirsch, "Analysis of the Rising Cost of Public Education," in: Joint Economic Committee of the Congress of the United States Government, 89th Congress, 1st Session. Study Paper, Washington, U. S. Government Printing Office, 1959, p. 26. M. Johnson and E. Scriver, "Class Size and Achievement Gains in Seventh and Eighth Grade English and Mathematics," in: School Quarterly, Autumn, 1967, p. 305.	This is often used as a proxy for teacher/student ratio. However, some, particularly Burkhead, and Johnson & Scriver say it is poor.

	National Education Association, <u>Research Bulletin</u> 39, #4, December, 1961.	
	Thomas, <u>op</u>. <u>cit</u>., p. 3.	
4. Teachers per 100 pupils.	Hirsch, <u>op</u>. <u>cit</u>., p. 27.	
5. Average pupil/teacher ratio.	Barron, <u>op</u>. <u>cit</u>., p. 291.	
	Burkhead, <u>op</u>. <u>cit</u>., p. 32.	
	J. Riew, "Economies of Scale in High School Operation," <u>Review of Economics and Statistics</u>, p. 282. August, 1966.	
6. Grouping.	Hirsch, <u>op</u>. <u>cit</u>., p. 26.	The grouping of common abilities would presumably be best measured by a dummy variable.
	Thomas, <u>op</u>. <u>cit</u>., p. 3.	
7. Teaching Load.	Hirsch, <u>op</u>. <u>cit</u>., p. 26.	
8. Average number of courses taught per teacher.	Riew, <u>op</u>. <u>cit</u>., p. 282.	

9.	College hours of average teacher.	Barron, _op_. _cit_., p. 291.
		Hirsch, _op_. _cit_., p. 27.
		P. R. Mort, W. S. Vincent and C. S. Newell, _The Growing Edge_. New York, Metropolitan School Study Council, 1963, p. 60.
10.	Percent teachers holding a Masters degree.	W. D. Firman, "The Relationship of Cost to Quality in Education," in: _Long-Range Planning in School Finance_, based on the Proceedings of the Sixth National School Finance Conference, St. Louis, 1963, p. 108.
		Hirsch, _op_. _cit_., p. 27.
		Riew, _op_. _cit_., p. 282.
11.	Professional training within the last three years.	Mort, _op_. _cit_., p. 59.
12.	Average teaching experience.	Barron, _op_. _cit_., p. 291.
		Riew, _op_. _cit_., p. 282.

		Thomas, _op_. _cit_., p. 3	
13.	Percent of teachers with permanent certificate.	Firman, _op_. _cit_., p. 109.	
14.	Percent of teachers with more than ten years experience.	Hirsch, _op_. _cit_., p. 27.	
15.	Percent of teachers with more than twenty years experience.	Firman, _op_. _cit_., p. 109.	**At this level experience had an inverse effect on quality.**
16.	Percent of teachers with two or more years experience.	W. I. Ackerman, "Teacher Competence and Pupil Changes," _Harvard Educational Review_, Vol. 24, No. 4. G. J. Anderson, _Input-Output Differences Among Secondary Schools_. Unpublished M. A. Thesis. McGill University -- MacDonald College, 1966.	

17.	Percent of teachers with five or more years experience.	C. H. Woollatt, The Cost-Quality Relationship on the Growing Edge. New York, Bureau of Publications, Teachers College, Columbia University, 1959.	
18.	Percent of teachers with one to four years experience.	Firman, op. cit., p. 109.	
19.	Average teacher salary.	Barron, op. cit., p. 191.	Woollatt uses this as a summary measure while Firman finds an inverse result.
		J. E. Cheal, Investment in Canadian Youth. Toronto, The MacMillan Company of Canada Limited, 1963, p. 110.	
		Firman, op. cit., p. 109.	
		Hirsch, op. cit., p. 27.	
		Riew, op. cit., p. 282.	
		Thomas, op. cit., p. 3.	
		Woollatt, op. cit.	
		Ayres, Laggards in Our Schools. Russell Sage Foundation, New York, 1909.	

20.	School retention rate.	Barron, op. cit., p. 290. Cheal, op. cit., p. 106. Shelley, as reported in Barron, op. cit., p. 281. Thomas, op. cit., p. 3.	Thomas emphasizes the size of the 12th grade.
21.	Teacher turnover.	Anderson, op. cit.	
22.	Teachers originally from outside the school district.	Mort, op. cit., p. 59.	Measures 21 - 24 indicate better quality results if boards actively hire outside state or even county boundaries.
23.	Percent of teachers who have lived in the community prior to employment as a teacher.	Firman, op. cit., p. 109.	
24.	Percent of teachers who earned undergraduate degrees in New York State colleges.	Firman, op. cit., p. 109.	

25.	Percent of teachers who earned graduate degrees in New York State colleges	Firman, _op. cit._, p. 109
26.	Number of high school credit units.	Hirsch, _op. cit._, p. 27. Riew, _op. cit._, p. 282.
27.	Number of functions carried out by the school.	Barron, _op. cit._, p. 291.
28.	Average amount of homework expected.	Thomas, _op. cit._, p. 3.
29.	Number of study halls per week .	Thomas, _op. cit._, p. 3.
30.	Number of books in school library.	Thomas, _op. cit._, p. 3.

31.	Age of school building.	Thomas, _op_. _cit_., p. 3.	
32.	Does school have a guidance program?	Thomas, _op_. _cit_., p. 3.	
33.	Percent of staff who have travelled a distance of at least 1200 miles (one way) away from home in the last 5 years at their own expense.	Firman, _op_. _cit_., p. 109.	
34.	Percent of college entrants.	Barron, _op_. _cit_., p. 290.	
35.	Individual attention in the classroom.	Barron, _op_. _cit_., p. 290.	This would require an intensive time study on all students.
36.	Number of special classes.	Barron, _op_. _cit_., p. 291.	

37. Provision of books and other instructional materials.	Barron, *op. cit.*, p. 290.

APPENDIX B

TITLES OF TESTS USED IN CARNEGIE STUDY

1. Canadian Academic Aptitude Tests (CAAT)
   (a) CAAT I - Verbal Reasoning
   (b) CAAT II - Mathematical Reasoning
   (c) CAAT III - Non-Verbal Reasoning
2. Canadian English Achievement Tests (CEAT)
   (a) CEAT I - Reading Achievement
   (b) CEAT II - Mechanics of Expression
   (c) CEAT III - Effectiveness of Expression
3. Canadian Mathematics Achievement Tests (CMAT)
   (a) CMAT I - Arithmetic Computation
   (b) CMAT II - Facts, Terms and Concepts
   (c) CMAT III - Measurement
4. Canadian Test of General Information (CTGI)
5. Canadian Achievement Test in French (CATF)
6. Canadian Achievement Test in English (CATE)
7. Canadian Achievement Test in Mathematics (CATM)
8. Canadian Physics Test (Ontario Edition) (CPTO)
9. Canadian Geometry Test (Ontario Edition) (CGTO)
10. Scholastic Aptitude Test - Verbal I
11. Scholastic Aptitude Test - Verbal II
12. Scholastic Aptitude Test - Verbal Total
13. Scholastic Aptitude Test - Mathematics
14. Achievement Test in French
15. Test in English Structure and Usage

APPENDIX C

COMPONENTS OF PROCESS MEASURES

Teachers	- Number of full time teachers employed by a school.
Students	- Average daily attendance.
Temporary Certificates	- Temporary Secondary School Certificate.
	Special Summer Course - Commercial Teachers.
	Special Summer Course - Technical Teachers.
	Occupational Trades Summer Course.
Letter of Permission	- A permit, held in lieu of formal qualification, to teach in a secondary school.
Basic Certificates	- Interim High School Assistant's Certificate, Type A.
	Interim High School Assistant's Certificate, Type B.
	Interim High School Assistant's Certificate, Type B (Endorsed).
	Permanent High School Assistant's Certificate.
	Permanent High School Assistant's Certificate (Endorsed).

APPENDIX C - continued

High School Specialist's Certificate.

High School Principal's Certificate.

Secondary School Principal's Certificate.

Interim Vocational Certificate, Type A.

Interim Vocational Certificate, Type B.

Permanent Vocational Certificate, Type B.

Vocational School Specialist's Certificate.

Vocational School Principal's Certificate.

Interim Occupational Certificate, Type A. Practical Subjects.

Interim Occupational Certificate, Type B. Practical Subjects.

Interim Occupational Certificate, Type A. General Subjects.

Interim Occupational Certificate, Type B. General Subjects.

APPENDIX C - continued

	Occupational Specialist's Certificate, Practical Subjects.
	Permanent Occupational Certificate, Practical Subjects.
	Occupational Specialist's Certificate, General Subjects.
	Permanent Occupational Certificate, General Subjects.
	Intermediate Industrial Arts Certificate.
	Specialist's Certificate in Industrial Arts.
Specialist Certificates	- Various certificates giving the holder specialist rating.

APPENDIX D

SUMMARY OF SECONDARY SCHOOL BOARDS

	Type of Board	Number of Boards
(a)	Boards having both Academic and Commercial & Vocational students.	187
(b)	Boards having Academic students only.	53
(c)	Boards having no students.[1]	116
(d)	Continuation school boards.[2]	19
(e)	Boards operating Junior High Schools as a separate entity.	1
(f)	Boards not reporting Academic and Commercial & Vocational activities separately.	1
	Total	377

[1] These boards are set up to collect fees for students living in their area of jurisdiction. Since no secondary schools are operated in these areas these fees are paid to the boards which operate the schools which the students attended.

[2] These are small boards which are gradually being phased out of existence. They serve primarily grades 9 and 10 in special areas. Data collected for these boards are not used in any of the following analysis.

APPENDIX E

BOARDS OPERATING SCHOOLS FOR ACADEMIC STUDENTS ONLY

School Size (ADA)	Number of Schools	Number of Schools Deleted
0-50		
51-100	3	2
101-150	11	4
151-200	9	3
201-250	8	3
251-300	8	2
301-350	7	1
351-400	2	Ø
401-450	1	Ø
451-500	2	1
501-550	1	1
551-600		
601-650		
651-700		
701-750		
751-800		
801-850		
851-900		
901-950		
951-1000		
1001-1050		
1051-1200	1	1
1201-1250		
	53	18

APPENDIX F

BOARDS OPERATING ONE SCHOOL ONLY - ACADEMIC STUDENTS

School Size (ADA)	Number of Schools	Number of Schools Deleted
0-50		
51-100		
101-150	3	1
151-200	6	4
201-250	14	4
251-300	12	4
301-350	15	6
351-400	15	6
401-450	16	6
451-500	8	Ø
501-550	9	3
551-600	10	3
601-650	3	Ø
651-700	5	2
701-750		
751-800	1	Ø
801-850	1	1
851-900	1	1
901-950		
951-1000		
1001-1050	1	1
	120	42

APPENDIX G

BOARDS OPERATING ONE SCHOOL ONLY - C&V STUDENTS

School Size (ADA)	Number of Schools	Number of Schools Deleted
0-50	17	7
51-100	25	8
101-150	13	4
151-200	9	3
201-250	9	6
251-300	10	3
301-350	9	3
351-400	5	1
401-450	10	4
451-500	4	2
501-550	2	Ø
551-600	1	Ø
601-650	1	Ø
651-700		
701-750	3	2
751-800	1	Ø
801-850		
851-900		
901-950		
951-1000	1	Ø
	120	43

APPENDIX H

BOARDS OPERATING MORE THAN ONE SCHOOL - ACADEMIC STUDENTS

School Size (ADA)	Number of Schools	Number of Schools Deleted	School Size (ADA)	Number of Schools	Number of Schools Deleted
351-400	1	Ø	1701-1750	1	Ø
401-450	1	Ø	1751-1800	2	1
451-500			1801-1850	2	1
501-550	2	Ø	1851-1900	1	Ø
551-600	2	1	1901-1950		
601-650	1	Ø	1951-2000		
651-700	2	Ø	2001-2050		
701-750	3	Ø	2051-2100		
751-800	3	2	2101-2150	2	Ø
801-850	1	1	2151-2200		
851-900	3	1	2201-2250		
901-950			2251-2300		
951-1000	1	Ø	2301-2350		
1001-1050	4	2	2351-2400	2	1
1051-1100			2401-2450		
1101-1150	2	Ø	2451-2500	1	Ø
1151-1200	6	4	2501-2550		
1201-1250			2551-2600		
1251-1300	1	1	2601-2650		
1301-1350			2651-2700		
1351-1400	1	Ø	2701-2750		
1401-1450	2	Ø	2751-2800		
1451-1500	2	Ø	2801-2850	2	Ø
1501-1550	2	1	over 2850	11	2
1551-1600	1	1			
1601-1650	1	Ø	TOTAL	67	19
1651-1700					

APPENDIX I

BOARDS OPERATING MORE THAN ONE SCHOOL - COMMERCIAL AND VOCATIONAL STUDENTS

School Size (ADA)	Number of Schools	Number of Schools Deleted	School Size (ADA)	Number of Schools	Number of Schools Deleted	School Size (ADA)	Number of Schools	Number of Schools Deleted
0-50	1	Ø	1001-1050	4	Ø	2001-2050	1	1
51-100	1	Ø	1051-1100	1	1	2051-2100	1	Ø
101-150			1101-1150	1	Ø	2101-2150		
151-200			1151-1200	1	Ø	2151-2200		
201-250	1	Ø	1201-1250	1	Ø	2201-2250		
251-300	3	1	1251-1300	1	Ø	2251-2300	1	1
301-350			1301-1350	1	Ø	2301-2350		
351-400	3	2	1351-1400			2351-2400		
401-450	6	1	1401-1450			2401-2450		
451-500	3	2	1451-1500			2451-2500		
501-550	1	Ø	1501-1550	4	1	2501-2550		
551-600	4	1	1551-1600			2551-2600		
601-650	3	2	1601-1650	1	Ø	2601-2650	1	Ø
651-700	1	Ø	1651-1700			2651-2700		
701-750	2	1	1701-1750			2701-2750	1	1
751-800	2	Ø	1751-1800	2	1	2751-2800	1	Ø
801-850	1	1	1801-1850	1	Ø	Over 2800	7	1
851-900	3	1	1851-1900					
901-950			1901-1950					
951-1000	1	Ø	1951-2000			TOTAL	67	19

APPENDIX J

THE COST-QUALITY NEXUS

In this study quality measures are used to make a correction on our output measure (Average Daily Attendance). The direct influence of quality on cost is not analysed. However, it is this latter question which is the main focus of attention in many educational cost studies. A positive, significant relationship is normally postulated and discovered. To see if this is true in the case of the public secondary school sub-sector in Ontario, the following structural form is estimated.[1]

$$AC = a\frac{1}{X} + b + cX + dX^2 + e\,Q_1 \quad ^2$$

$$AC = a\frac{1}{X} + b + cX + dX^2 + e\,Q_2$$

where Q_1 = Index A

Q_2 = Index B

---

[1] The private and separate sub-sectors are omitted because of their small size.

[2] The form $TC = a + bX + cX^2 + dX^3 + e\,Q$ was also estimated but, in general, the Q's were not significant. It has been suggested that this may be because the large values of TC and the X's were 'swamping' the relatively small values of the Q's.

APPENDIX J - continued

The following are the relevant equations:[3]

For 161 academic boards:

$$AC = \underset{\substack{(28.7)\\ [19.4]}}{557.7} + \underset{\substack{(26.9)\\ [3.3]}}{88Q_1} \qquad \text{(J-1)}$$

SE = 127 $\qquad R^2 = .07601$

$$AC = \underset{\substack{(80.98)\\ [6.34]}}{513.6} + \underset{\substack{(79.49)\\ [1.65]}}{130.7Q_2} \qquad \text{(J-2)}$$

SE = 130 $\qquad R^2 = .01673$

For 125 Commercial and Vocational boards:

$$AC = \underset{\substack{(995.5)\\ [2.38]}}{\frac{2366}{X}} + \underset{\substack{(125.3)\\ [3.65]}}{457.8} + \underset{\substack{(.035)\\ [4.49]}}{.16X} - \underset{\substack{(.0000059)\\ [-3.59]}}{.00002X^2} + \underset{\substack{(117.5)\\ [2.22]}}{261.1Q_2} \qquad \text{(J-3)}$$

SE = 144 $\qquad R^2 = .18329$

Q_1 is not significant

---

[3] Only equations having significant variables are exhibited.

APPENDIX J - continued

For 113 boards operating an academic programme and one school only we get insignificant results. This is also the case for the 77 boards operating a commercial and vocational programme and one school only. It should be noted, however, that in all cases where the Q's are insignificant they are of the correct (positive) sign.

In the Ontario case then we find some support for the cost-quality nexus. In addition, where the equations have significant coefficients the shape of the resulting average cost curve is of the same general type as that yielded using total costs. This means that similar results emerge when quality is used additively and multiplicatively.

APPENDIX K

AVERAGE COST EQUATIONS

In this Appendix we present the results of fitting the structural form $AC = a\frac{1}{X} + b + cX + dX^2$ to our cost data. Only equations having significant coefficients are exhibited. It should be noted that in general the equations estimated in this method have higher coefficients of variation than those estimated by the structural form $TC = a + bX + cX^2 + dX^3$.

For 161 academic boards the results are:

$$AC_1 = \underset{\substack{(10.33)\\[62.49]}}{645.7}{}^{1} \qquad \text{(K-1)}$$

SE = 131 $\qquad$ R^2 = (See Footnote #2)

$$AC_2 = \underset{\substack{(453)\\[267]}}{\frac{120900}{X_2}} + \underset{\substack{(27.62)\\[6.05]}}{167} + \underset{\substack{(.029)\\[7.62]}}{.22X_2} - \underset{\substack{(.0000025)\\[-5.79]}}{.000015X_2^2} \qquad \text{(K-2)}$$

SE = 252 $\qquad$ $R^2 = .99781$

---

1 The subscripted variables have the same meaning as they do when they appear in the dummy variable analysis in Chapter IV.

2 When the equation is estimated using only a constant the programme does not compute this statistic. Equations (K-7), (K-10), (K-13) are also effected.

APPENDIX K

AVERAGE COST EQUATIONS

In this Appendix we present the results of fitting the structural form $AC = a\frac{1}{X} + b + cX + dX^2$ to our cost data. Only equations having significant coefficients are exhibited. It should be noted that in general the equations estimated in this method have higher coefficients of variation than those estimated by the structural form $TC = a + bX + cX^2 + dX^3$.

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---

[1] The subscripted variables have the same meaning as they do when they appear in the dummy variable analysis in Chapter IV.

[2] When the equation is estimated using only a constant the programme does not compute this statistic. Equations (K-7), (K-10), (K-13) are also effected.

APPENDIX K - continued

$$AC_3 = \frac{98560}{X_3} + 393.6 \qquad (K\text{-}3)$$

$$\begin{array}{cc} (4605) & (30.79) \\ [21.4] & [12.78] \end{array}$$

SE = 311 $\qquad R^2 = .74231$

For 125 Commercial and Vocational Boards the results are:

$$AC_1 = \frac{2004}{X_1} + 731.1 + .15X_1 - .00002X_1^2 \qquad (K\text{-}4)$$

$$\begin{array}{cccc} (998) & (24.48) & (.036) & (000006) \\ [2.01] & [29.87] & [4.22] & [3.33] \end{array}$$

SE = 147 $\qquad R^2 = .14967$

$$AC_2 = \frac{6324}{X_2} + 667.8 + .19X_2 - .000026X_2^2 \qquad (K\text{-}5)$$

$$\begin{array}{cccc} (1002) & (30.32) & (.046) & (.0000076) \\ [6.31] & [22.02] & [4.25] & [-3.41] \end{array}$$

SE = 192 $\qquad R^2 = .27188$

$$AC_3 = \frac{11000}{X_3} + 820.2 \qquad (K\text{-}6)$$

$$\begin{array}{cc} (2724) & (85.18) \\ [9.63] & [4.04] \end{array}$$

SE = 877 $\qquad R^2 = .11706$

APPENDIX K - continued

The 113 boards operating an academic programme and one school only produce the following results:

$$\underset{\substack{(11.87)\\[53.37]}}{AC_1 = 633.3} \qquad \text{(K-7)}$$

SE = 126

$$AC_2 = \underset{\substack{(279.7)\\[437.8]}}{\frac{122600}{X_2}} - \underset{\substack{(70.99)\\[-9.59]}}{680.6} + \underset{\substack{(.38)\\[10.22]}}{3.89X_2} - \underset{\substack{(.00047)\\[-7.33]}}{.003X_2^2} \qquad \text{(K-8)}$$

SE = 138 $\qquad R^2 = .99954$

$$AC_3 = \underset{\substack{(5674)\\[18.29]}}{\frac{103800}{X_3}} + \underset{\substack{(45.0)\\[7.19]}}{323.6} \qquad \text{(K-9)}$$

SE = 350 $\qquad R^2 = .75085$

For the 77 boards operating a Commercial and Vocational programme and one school only, the results are:

APPENDIX K - continued

$$AC_1 = \underset{\substack{(16.96) \\ [46.16]}}{782.9} \qquad \text{(K-10)}$$

SE = 149

$$AC_2 = \underset{\substack{(987.5) \\ [1.78]}}{\frac{1753}{X_2}} + \underset{\substack{(20.78) \\ [35.58]}}{739.4} \qquad \text{(K-11)}$$

SE = 149 $\qquad R^2 = .04034$

$$AC_3 = \underset{\substack{(3557) \\ [2.80]}}{\frac{9955}{X_3}} + \underset{\substack{(140.3) \\ [6.33]}}{888.5} \qquad \text{(K-12)}$$

SE = 1093 $\qquad R^2 = .09456$

Finally, the 14 schools in the private and separate sub-sectors produce the following results:

$$AC_1 = \underset{\substack{(168.9) \\ [6.24]}}{1053} \qquad \text{(K-13)}$$

SE = 609

APPENDIX K - continued

$$AC_2 = 822.3 + \frac{47470}{X_2} \qquad \text{(K-14)}$$

$$\begin{array}{ll} (181.2) & (322.7) \\ [4.54] & [147.1] \end{array}$$

SE = 652 $\qquad R^2 = .99945$

In eight cases the average cost equations produce similar results to the total cost equations. The exceptions are as follows. (a) The raw data for the 161 Academic boards produces a 'rotated J' cost curve in the case of total costs and constant costs when average cost is the dependent variable. (b) For the same 161 boards, Index A-weighted data produce a 'rotated J' average cost curve when total cost is the dependent variable and a 'Lazy S' curve when average cost is used. (c) When Index A-weighted data is used in the case of the 113 Academic boards operating one school only the total cost equation produces constant costs while the average cost equation produces declining costs. (d) In the case of the 77 Commercial & Vocational boards operating one school only the raw data produces an average cost curve shaped like an inverted 'U' when total cost is the dependent variable and constant costs when average cost is used.

APPENDIX K - continued

(e) When Index A-weighted data is used for the same boards, total cost produces a declining cost curve while average cost produces a curve shaped like an inverted 'U'. (f) The raw data in the private and separate sub-sectors produce declining average costs when total cost is the dependent variable and constant average costs when the dependent variable is average cost.

Each of these cases are discussed more fully in the relevant sections of Chapter IV.

APPENDIX L

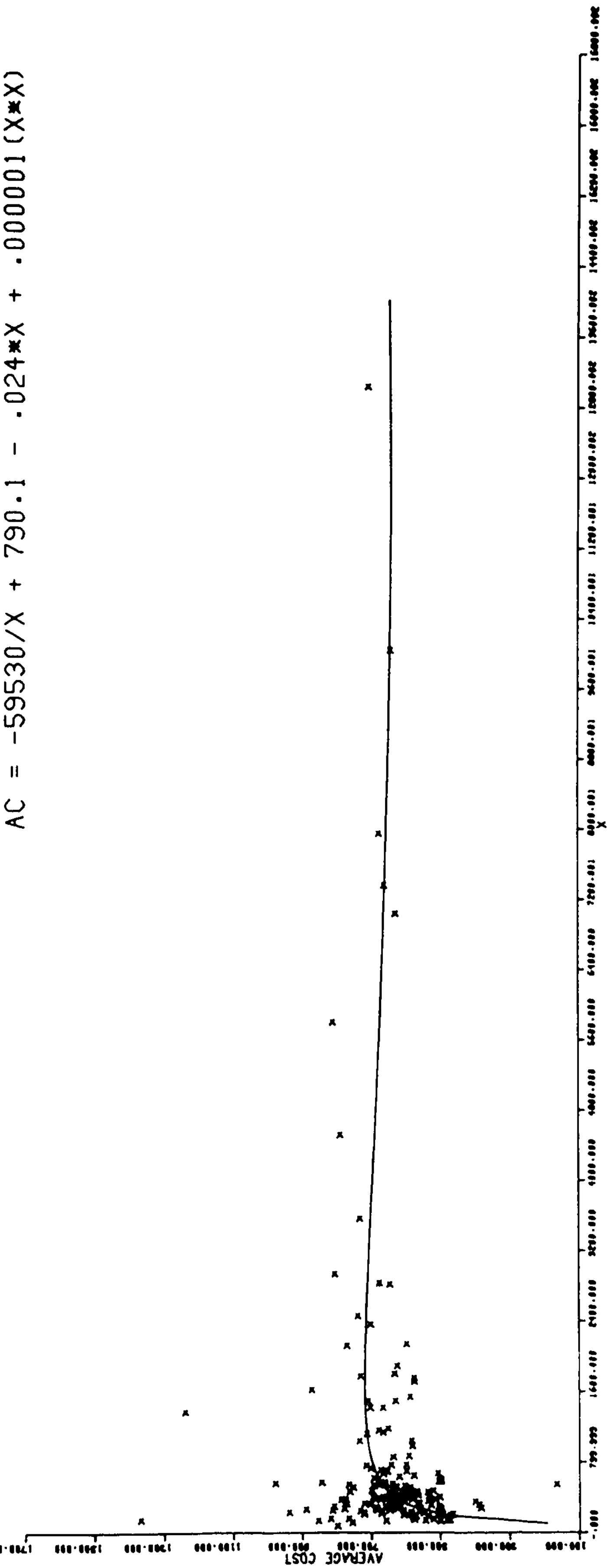
SUMMARY STATISTICS FOR 161 ACADEMIC BOARDS

Item	Mean
Administrative Salaries	$ 17,120
Administrative Total	30,932
Plant Operation and Maintenance	69,390
Instructional Salaries	367,183
Instructional Supplies	32,803
Interest on Temporary Borrowing	1,677
Long Term Interest Charges	33,298
Property Tax	60,344
Sales Tax	561
Depreciation	34,817
Average Daily Attendance	949

ACADEMIC - ALL SCHOOL BOARDS

SCATTER AND FITTED EQUATION - RAW DATA

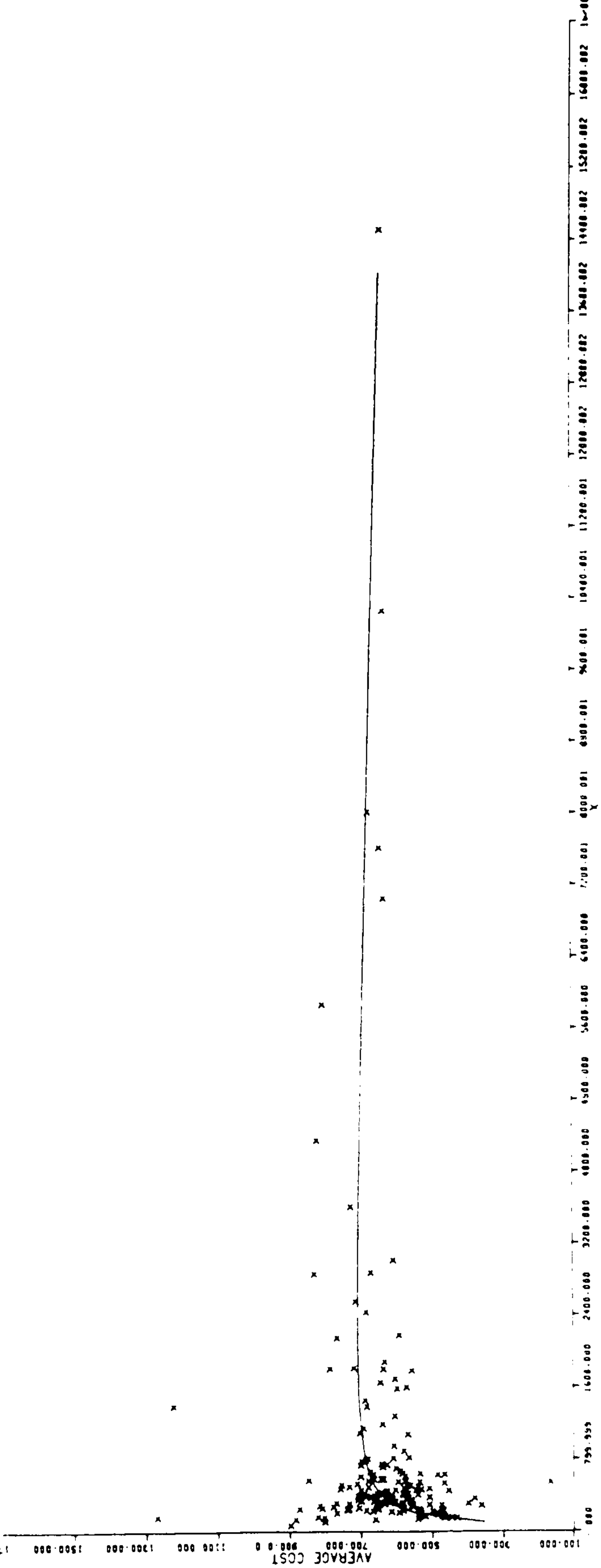
$$AC = -59530/X + 790.1 - .024*X + .000001(X*X)$$



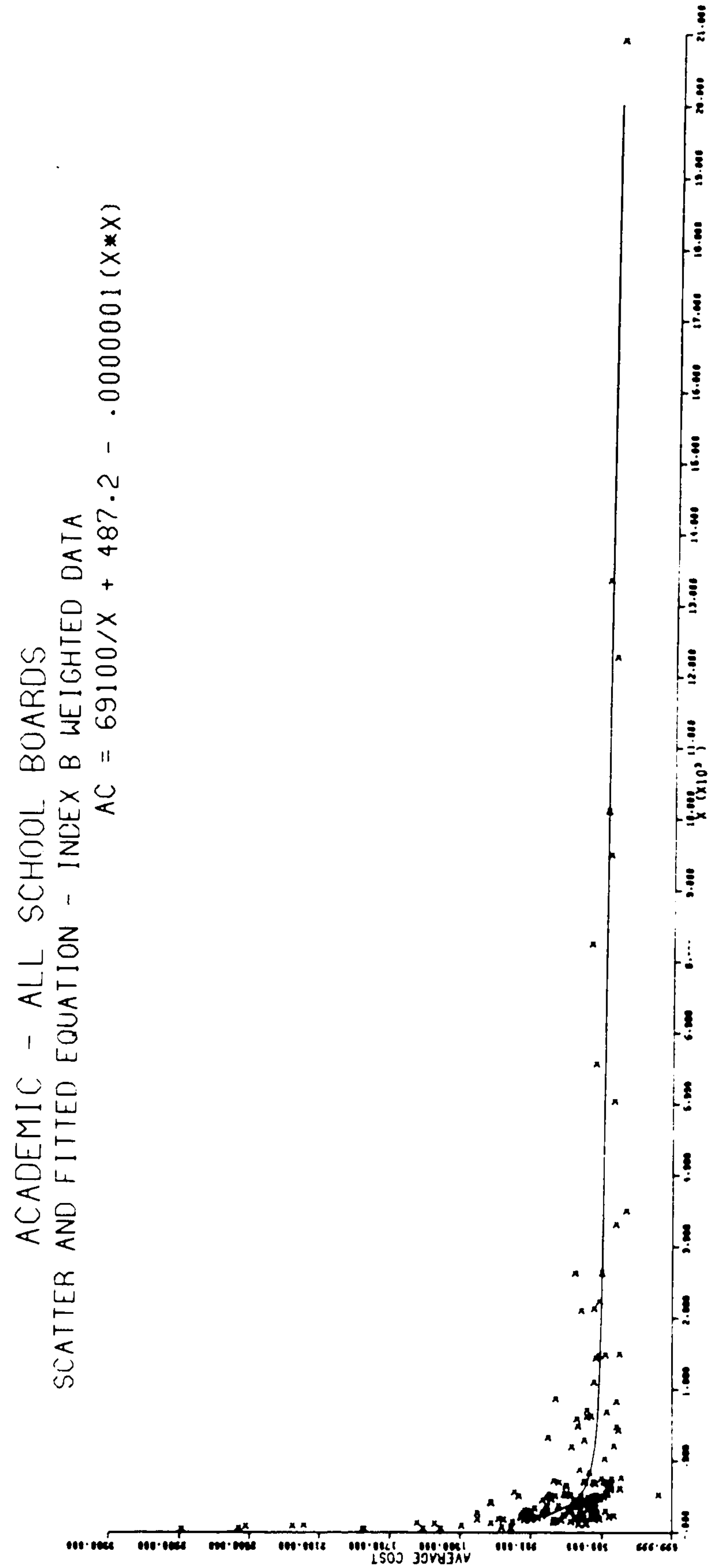
ACADEMIC ALL SCHOOL BOARDS

SCATTER AND FITTED EQUATION - INDEX A WEIGHTED DATA

AC = -38840/X + 731.9 - .007*X



APPENDIX O


ACADEMIC - ALL SCHOOL BOARDS
SCATTER AND FITTED EQUATION - INDEX B WEIGHTED DATA
AC = 69100/X + 487.2 - .0000001(X*X)
AVERAGE COST
X (X10³)

APPENDIX P

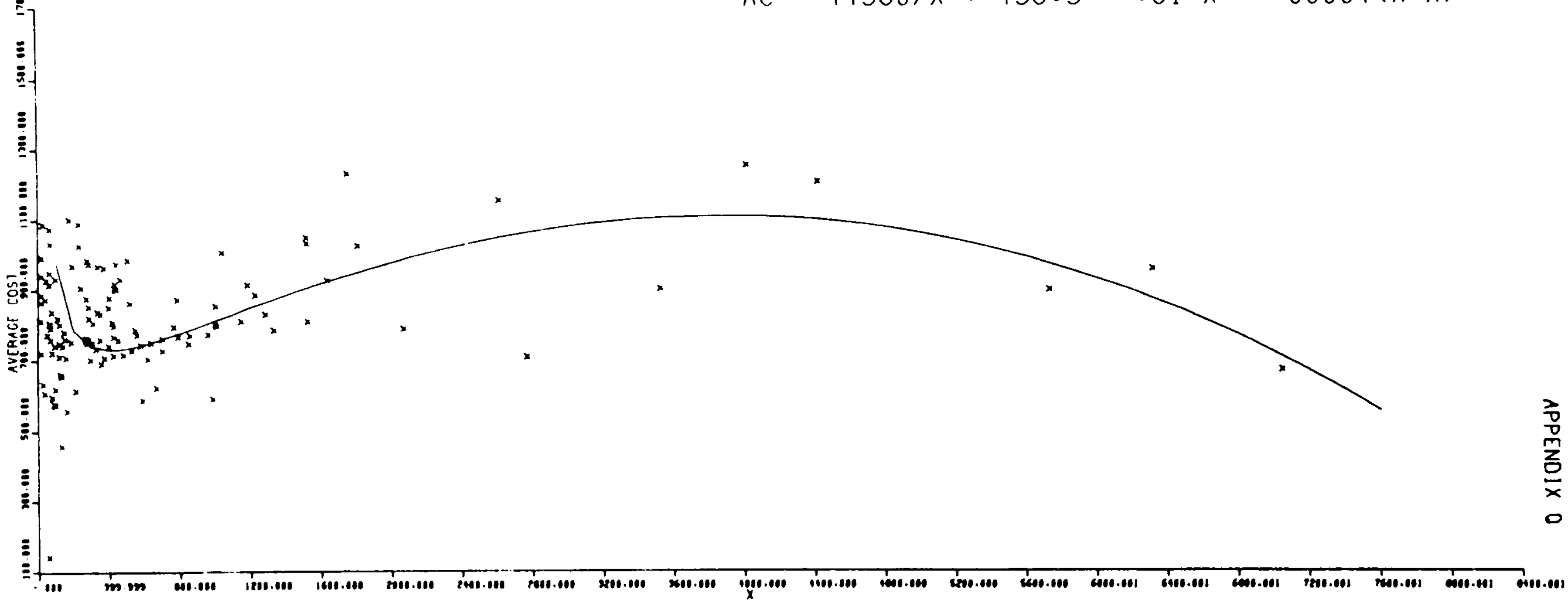
SUMMARY STATISTICS FOR 125 COMMERCIAL & VOCATIONAL BOARDS

Item	Mean
Administrative Salaries	$ 16,264
Administrative Total	27,994
Plant Operation and Maintenance	69,111
Instructional Salaries	312,242
Instructional Supplies	35,778
Interest on Temporary Borrowing	1,263
Long Term Interest Charges	21,569
Property Tax	95,257
Sales Tax	612
Depreciation	53,146
Average Daily Attendance	708

C + V - ALL SCHOOL BOARDS

SCATTER AND FITTED EQUATION - RAW DATA

AC = 44980/X + 496.9 + .31*X - .00004(X*X)

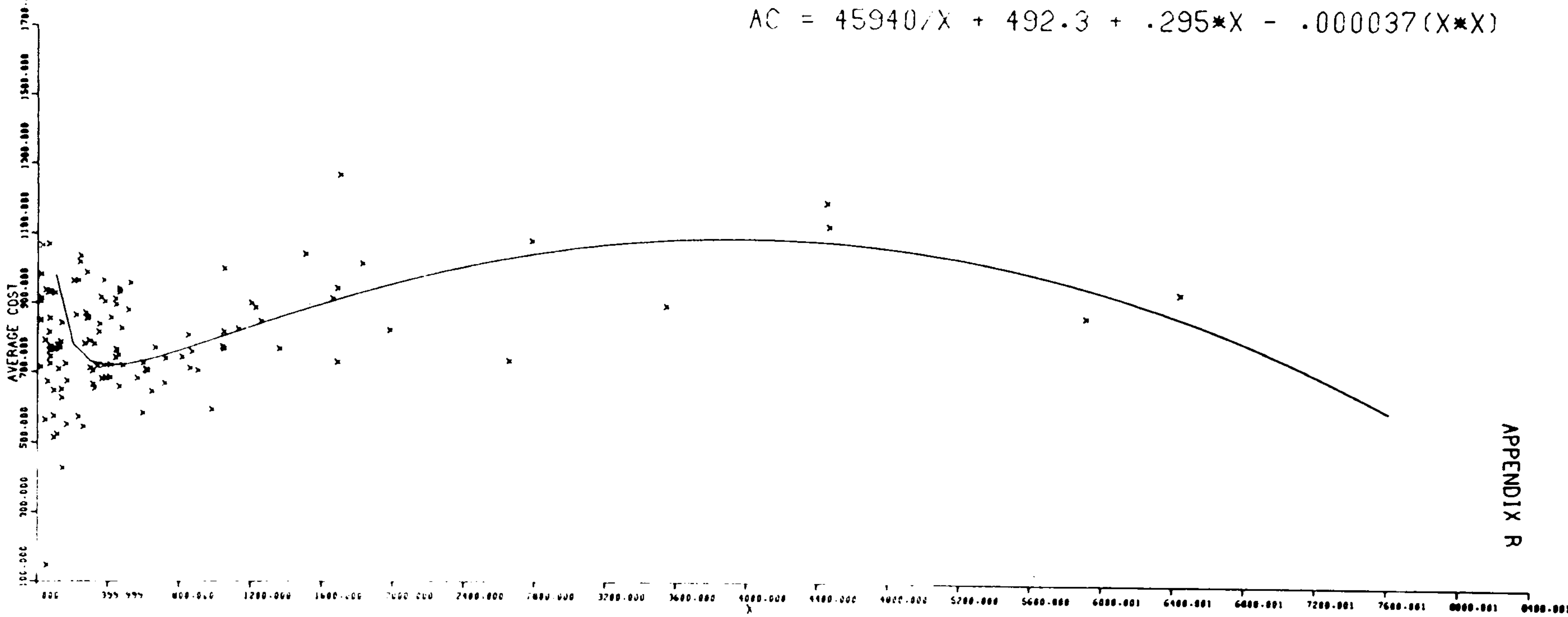


APPENDIX R

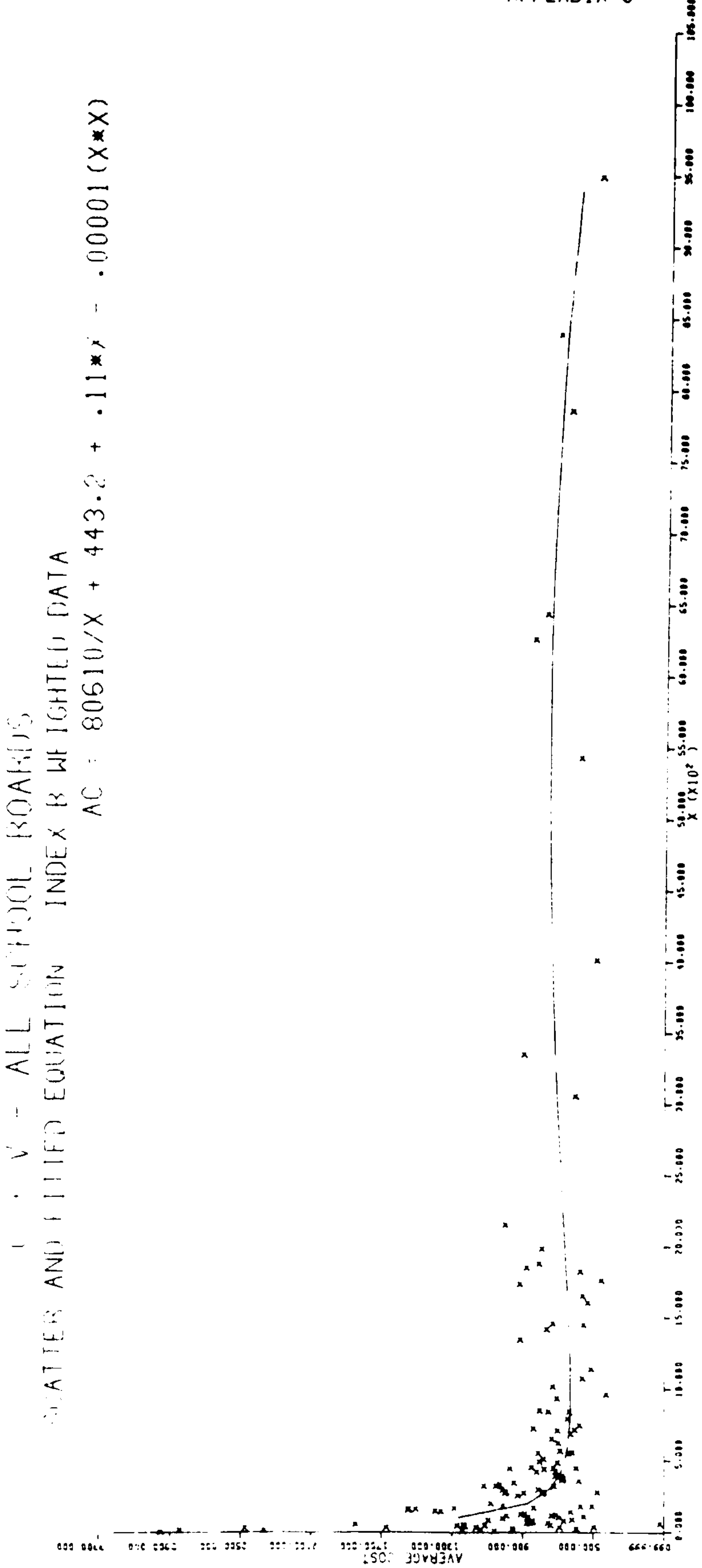
C + V - ALL SCHOOL BOARDS

SCATTER AND FITTED EQUATION - INDEX A WEIGHTED DATA

AC = 45940/X + 492.3 + .295*X - .000037(X*X)



APPENDIX S

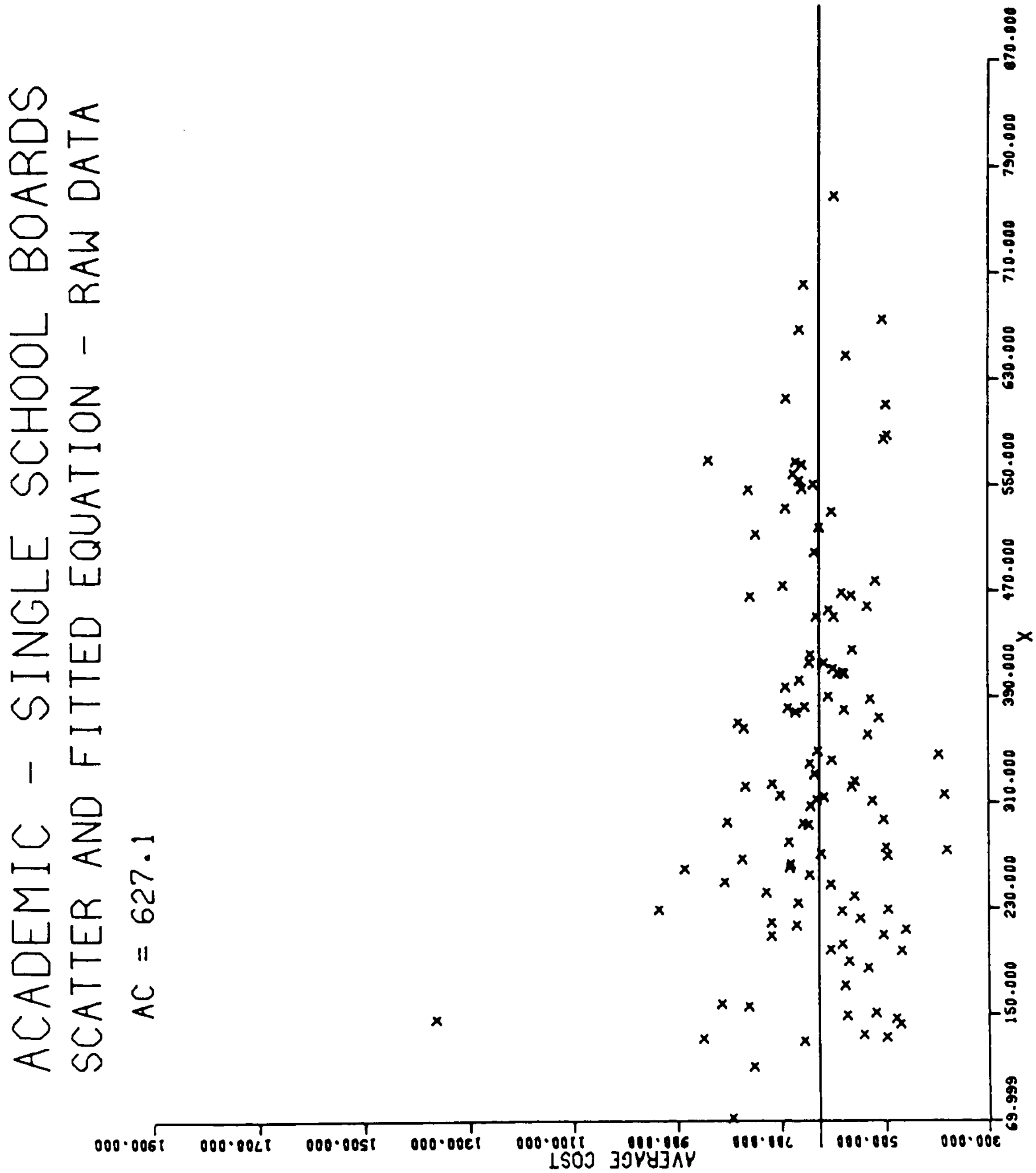

ALL SCHOOL BOARDS
AND FITTED EQUATION INDEX B WEIGHTED DATA
AC = 80610/X + 443.2 + .11*X - .00001(X*X)
AVERAGE COST
X (X10²)

APPENDIX T

SUMMARY STATISTICS FOR 113 ACADEMIC BOARDS

Item	Mean
Administrative Salaries	$ 5,321
Administrative Total	9,719
Plant Operation and Maintenance	19,928
Instructional Salaries	126,089
Instructional Supplies	12,135
Interest on Temporary Borrowing	1,252
Long Term Interest Charges	14,481
Property Tax	22,377
Sales Tax	208
Depreciation	12,205
Average Daily Attendance	352

APPENDIX U



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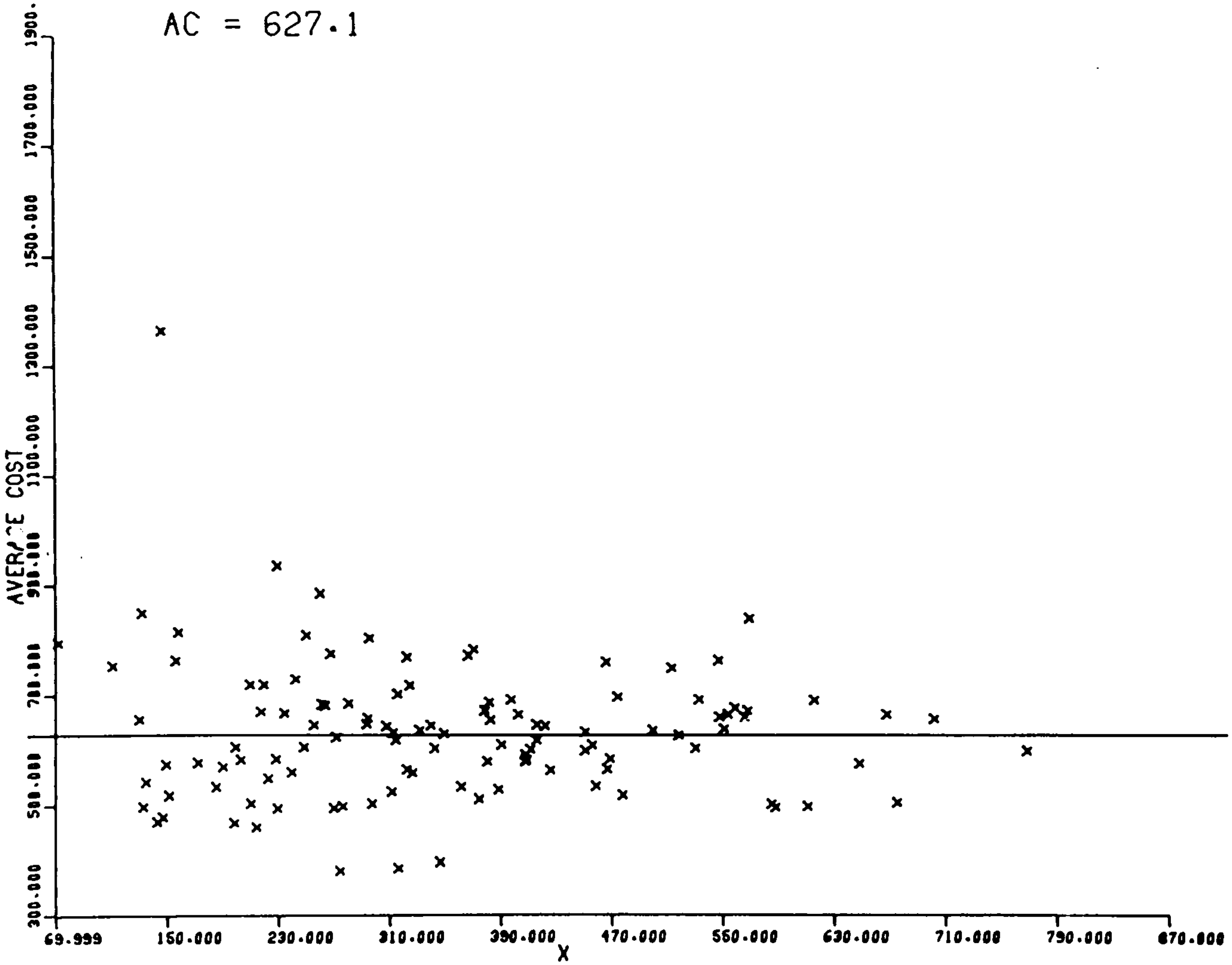
APPENDIX U

ACADEMIC - SINGLE SCHOOL BOARDS

SCATTER AND FITTED EQUATION - RAW DATA



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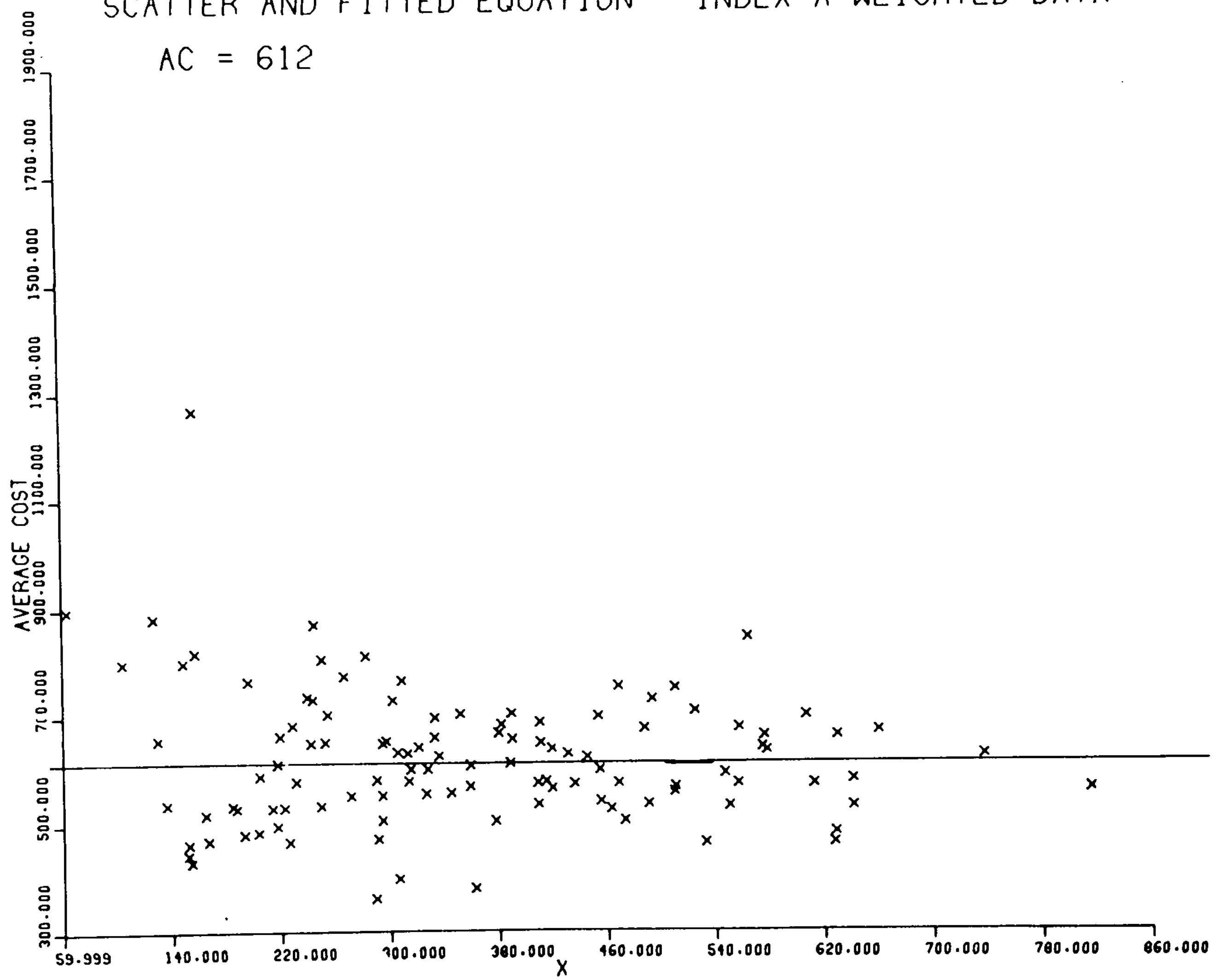
APPENDIX V

ACADEMIC - SINGLE SCHOOL BOARDS

SCATTER AND FITTED EQUATION - INDEX A WEIGHTED DATA

AC = 612



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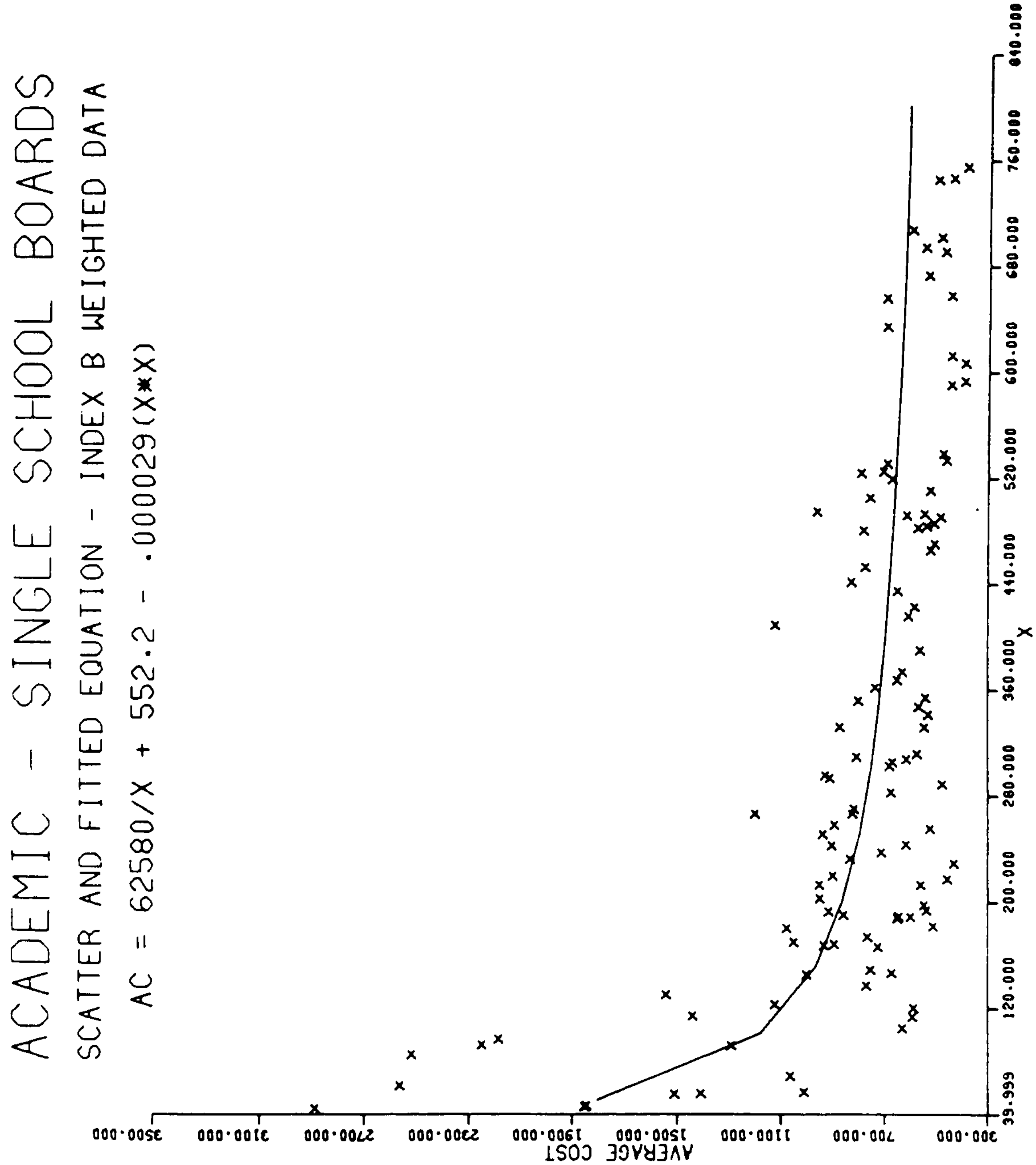
APPENDIX W


ACADEMIC - SINGLE SCHOOL BOARDS
SCATTER AND FITTED EQUATION - INDEX B WEIGHTED DATA
AC = 62580/X + 552.2 - .000029(X*X)
AVERAGE COST
300.000
700.000
1100.000
1500.000
1900.000
2300.000
2700.000
3100.000
3500.000
39.999
120.000
200.000
280.000
360.000
440.000
520.000
600.000
680.000
760.000
840.000

APPENDIX X

SUMMARY STATISTICS FOR 77 COMMERCIAL & VOCATIONAL BOARDS

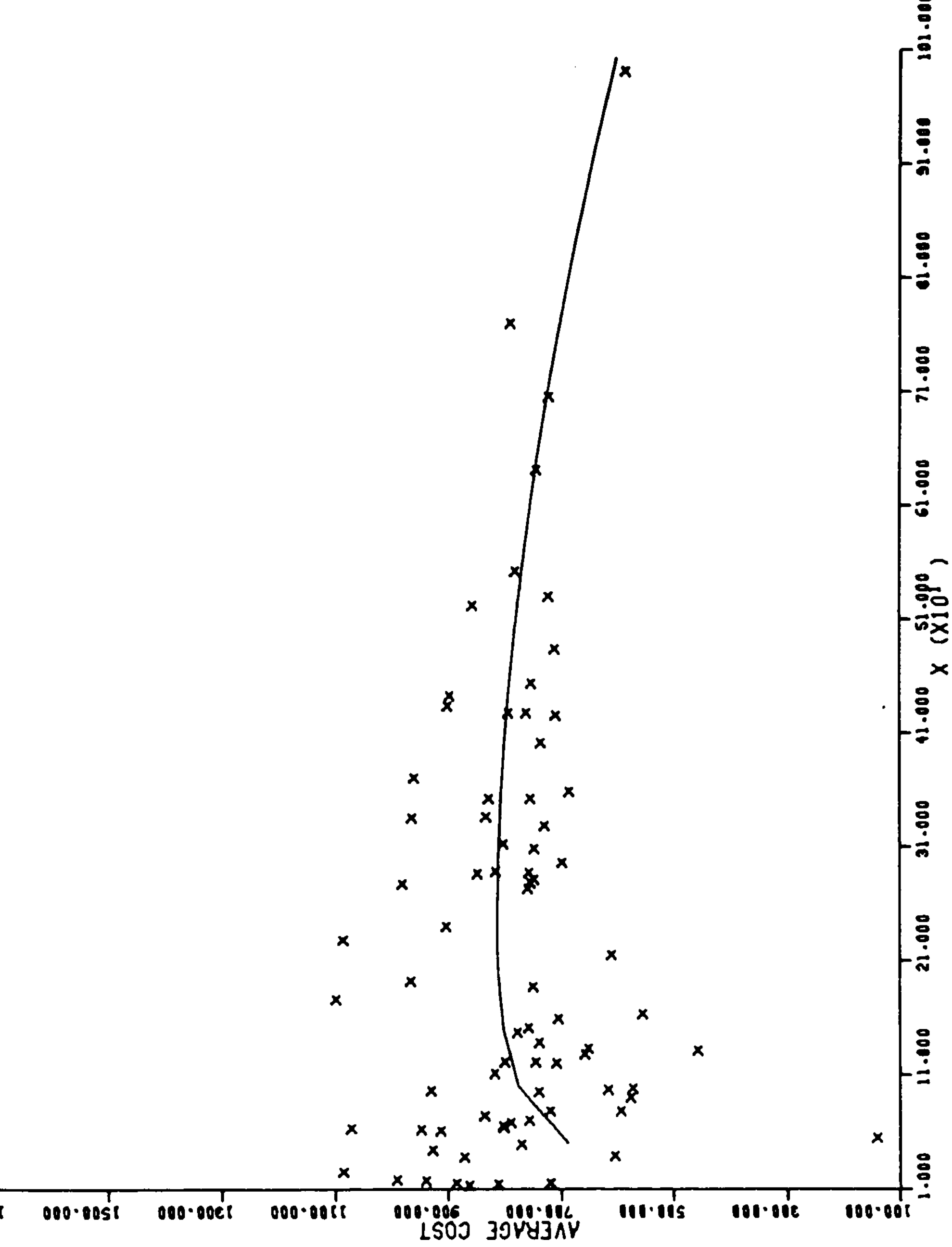
Item	Mean
Administrative Salaries	$ 3,870
Administrative Total	6,882
Plant Operation and Maintenance	17,164
Instructional Salaries	91,960
Instructional Supplies	10,036
Interest on Temporary Borrowing	933
Long Term Interest Charges	4,706
Property Tax	30,840
Sales Tax	172
Depreciation	16,665
Average Daily Attendance	232

APPENDIX Y

C + V - SINGLE SCHOOL BOARDS

SCATTER AND FITTED EQUATION - RAW DATA

AC = -9000/X + 866.2 - .00025(X*X)



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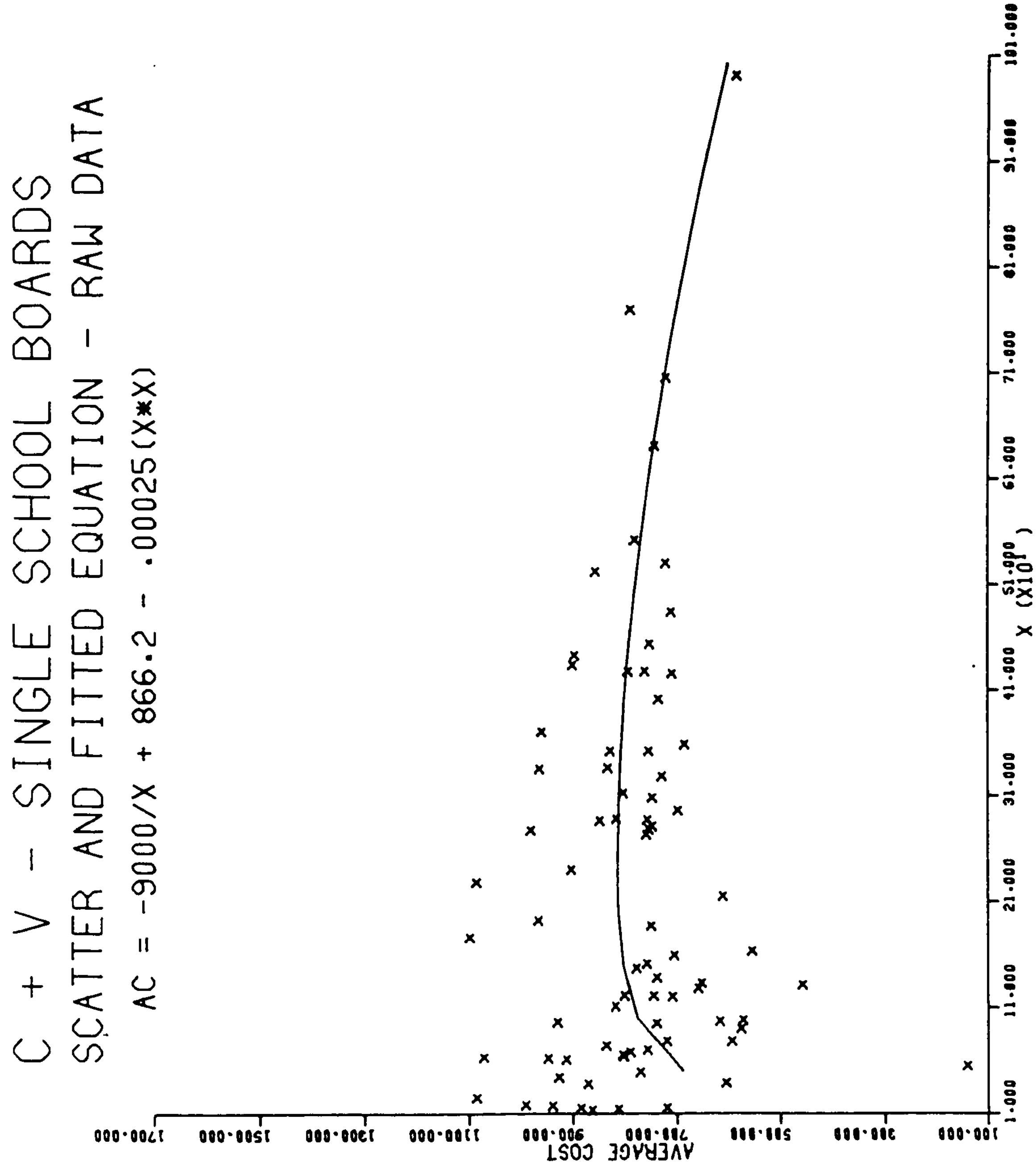
APPENDIX Y



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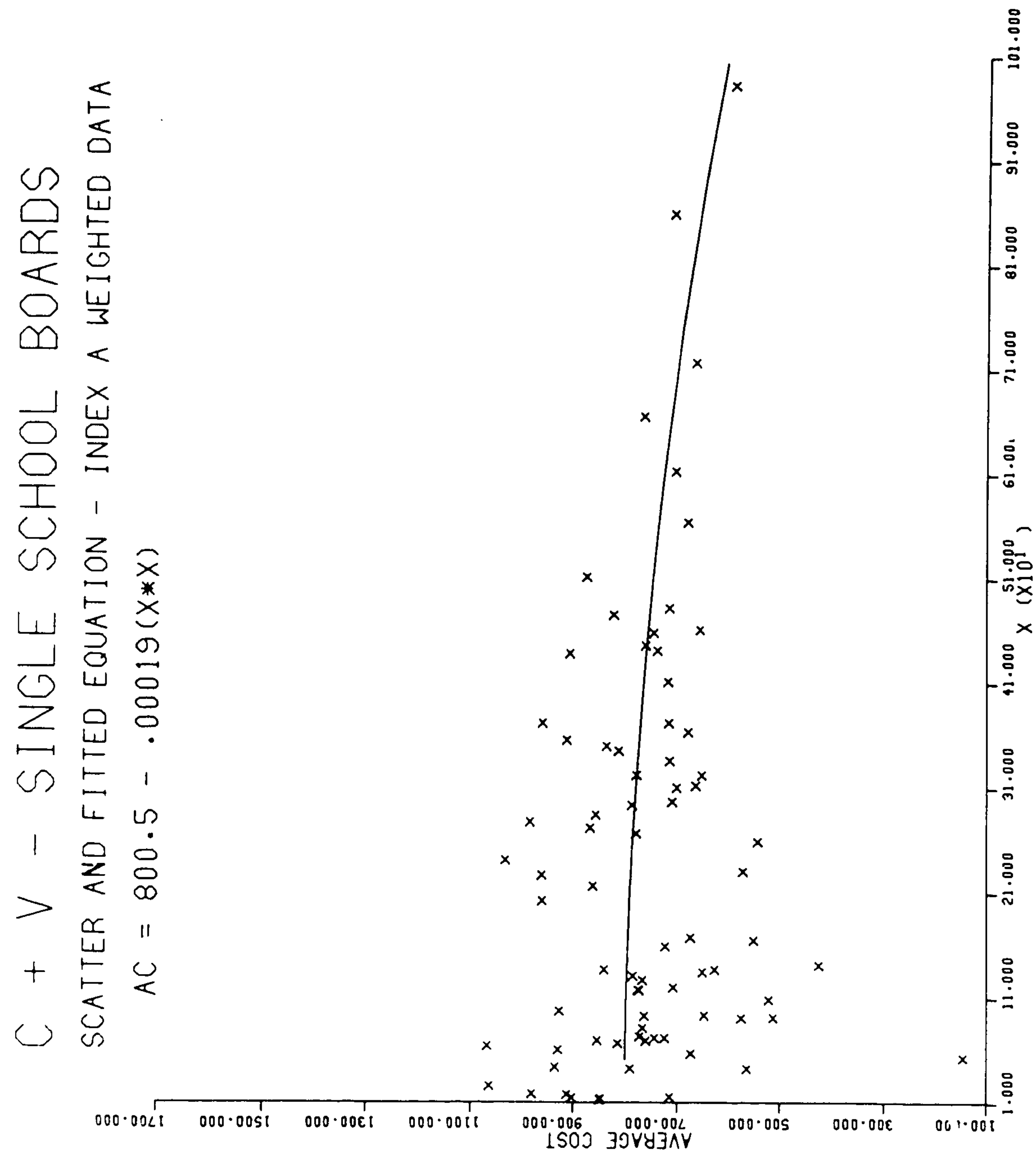
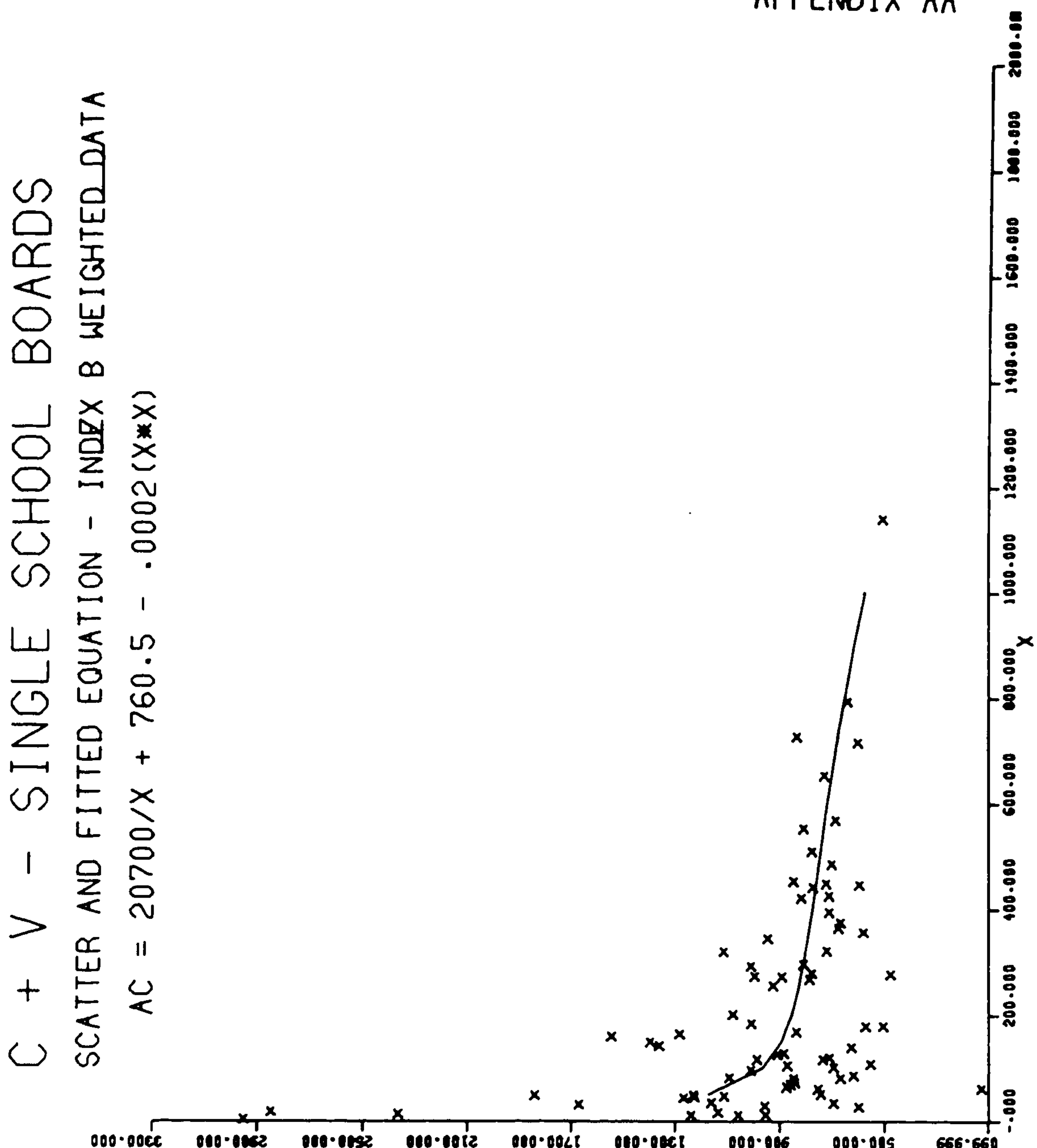
APPENDIX Z


C + V - SINGLE SCHOOL BOARDS
SCATTER AND FITTED EQUATION - INDEX A WEIGHTED DATA
AC = 800.5 - .00019(X*X)
AVERAGE COST
100.100
300.000
500.000
700.000
900.000
1100.000
1300.000
1500.000
1700.000
X (X10¹)
1.000
11.000
21.000
31.000
41.000
51.000
61.000
71.000
81.000
91.000
101.000

APPENDIX AA


C + V - SINGLE SCHOOL BOARDS
SCATTER AND FITTED EQUATION - INDEX B WEIGHTED DATA
AC = 20700/X + 760.5 - .0002(X**X)
AVERAGE COST
X

APPENDIX BB

QUESTIONNAIRE MAILED TO SEPARATE SCHOOLS

Page 1.

INSTRUCTIONS FOR COMPLETION OF DATA SHEET

1. Please use data for the year 1964 only. If your accounts are kept for the period of a school year (September-June), please submit data for the 1963-64 term.

2. The enclosed envelope is for your convenience.

APPENDIX BB - continued

Page 2.

DATA SHEET

1. Name of School ______________________________

2. Financial data is actual ____ or estimated ____ (Check one)

3. FINANCIAL DATA

Administrative Salaries	______________
Other Administrative Expenses	______________
Plant Operation and Maintenance (excluding salaries))	______________
Salaries for Plant Operation and Maintenance	______________
Instructional Salaries	______________
Library Cost	______________
Cost of Extracurricular Activities	______________
Bursaries	______________
Interest on Temporary Borrowing	______________
Capital Expenditures from Currents Funds	______________
Other Expenses (please explain)	
______________________________	
______________________________	
______________________________	______________

APPENDIX BB - continued

Page 3.

Long-term debt charges - Interest ______________

Principal Installments ______________

Sinking Fund Requirements ______________

Depreciation ______________

Other long-term expenses (please explain)

____

____

______________________________ ______________

Tuition Fees Received ______________

Government Grants ______________

Funds received from Parish ______________

Other Sustaining Funds (please explain)

____

____

______________________________ ______________

Value of Fixed Assets at Cost ________ or Present Value _____
(please check one)

Land ______________

Buildings ______________

APPENDIX BB - continued

Page 4.

Furniture & Equipment	______________
Projects in Progress	______________
Other Capital Assets	______________
Amount of Endowment	______________

4. NON-FINANCIAL DATA

Average Daily Attendance	______________
Enrolment	______________
No. of Teachers in 1964	______________
No. of Teachers in 1963	______________
No. of Teachers with B.A.	______________
No. of Teachers with M.A.	______________
No. of Teachers with Ph.D.	______________
No. of Teachers with 5 or more years' experience	______________
No. of Teachers with 2 or more years' experience	______________
No. of Teachers employed in 1963 who remained on staff in 1964	______________

APPENDIX CC

QUESTIONNAIRE MAILED TO INDEPENDENT SCHOOLS

Page 1.

INSTRUCTIONS FOR COMPLETION OF DATA SHEET

1. Please use data for the year 1964 only. If your accounts are kept for the period of a school year (September-June), please submit data for the 1963-64 term.
2. Please note that only information on secondary education is required. If you do not account separately for your secondary operations, please estimate these costs and indicate that they are estimated.
3. The enclosed envelope is for your convenience.

APPENDIX CC - continued

Page 2.

DATA SHEET

1. Name of School ________________________

2. Financial data is actual ____ or estimated ____ (check one)

3. FINANCIAL DATA

Administrative Salaries	________
Other Administrative Expenses	________
Plant Operation and Maintenance (excluding salaries and boarding costs)	________
Salaries for Plant Operation and Maintenance	________
Boarding Costs	________
Instructional Salaries	________
Library Cost	________
Cost of Extracuricular Activities	________
Bursaries	________
Interest on Temporary Borrowing	________
Capital Expenditures from Current Funds	________
Taxes	________
Other Expenses (please explain)	
________________________	
________________________	
________________________	________

APPENDIX CC - continued

Page 3.

Long-term debt charges - Interest ____________

Principal Installments ____________

Sinking Fund Requirements ____________

Depreciation ____________

Other long-term Expenses (please explain)

____

____

____________ ____________

Fees Received

Day Students ____________

Boarding Students ____________

Government Grants ____________

Other Sustaining Funds (please explain)

____

____

____________ ____________

Alumni Support ____________

Value of Fixed Assets at Cost ______ or Present Value

(please check one) ______

Land ____________

Buildings ____________

APPENDIX CC - continued

Page 4.

Furniture & Equipment	______________
Projects in Progress	______________
Other Capital Assets	______________
Amount of Endowment	______________

4. NON-FINANCIAL DATA

Average Daily Attendance	______________
Enrolment	______________
No. of Teachers in 1964	______________
No. of Teachers in 1963	______________
No. of Teachers with B.A.	______________
No. of Teachers with M.A.	______________
No. of Teachers with Ph.D.	______________
No. of Teachers with 5 or more years' experience	______________
No. of Teachers with 2 or more years' experience	______________
No. of Teachers employed in 1963 who remained on staff in 1964.	______________

APPENDIX DD

The University of Western Ontario, London, Canada

University College
Department of Economics

August 26, 1968

Dear

I am a graduate student in the Department of Economics at the University of Western Ontario. At present I am working on my Ph.D. thesis entitled "An Economic Analysis of the Secondary Education Sector in Ontario."

As part of my thesis I plan to analyse the independent secondary schools. In particular, I want to find out if economies of scale exist in the independent sector, i.e., does cost per student decrease as the size of the school increases.

In order to carry out this project I will need your assistance.

I would appreciate it if you would complete the attached form with the pertinent data for the year 1964. Naturally, this information will remain completely confidential.

If you have any questions about the information required, or if you desire an amplification of my purpose, I will reply immediately to your request.

I have included a copy of my curriculum vitae so that you may know someth g of my background. I have also included a letter from my thesis director concerning the use to which this data will be put.

Thank you for your consideration.

Sincerely,

Donald A. Dawson

Encls.

The University of Western Ontario, London, Canada

University College
Department of Economics

August 26, 1968

To Whom It May Concern:

This is to certify that Donald A. Dawson is a Ph. D. candidate working on his doctoral thesis under my direction.

Please be assured that any information which he obtains from you will be used for scholarly purposes only and will be presented in such a manner that no specific data can be associated with the school from which it was obtained.

Thank you for your cooperation.

Sincerely

R. J. Wonnacott
Acting Head

The University of Western Ontario, London, Canada

University College
Department of Economics

October 30, 1968

Dear

Approximately eight weeks ago I mailed you a questionnaire requesting financial data which, as I explained, would be used for my doctoral thesis. I realize that perhaps my timing in mailing this questionnaire was poor in that it probably arrived just at the busiest time of the year, when administrative duties connected with opening and registration must have been at their peak.

I trust that your work load has eased somewhat and I am hoping that I may expect to hear from you some time in the near future so that I can continue to compile the data so necessary for my study. Should my questionnaire and covering letters have gone astray in the mail, or should you require any additional information on my project, I would be most happy to supply either one.

Thanking you for your consideration of my request, I remain,

Yours sincerely,

Donald A. Dawson

APPENDIX GG

SUMMARY STATISTICS 14 INDEPENDENT AND SEPARATE SCHOOLS

Item	Mean
Administrative Salaries	$ 13,172
Administrative Total	28,620
Plant Operation and Maintenance	72,954
Instructional Salaries	91,390
Instructional Supplies	4,292
Interest on Temporary Borrowing	1,999
Long Term Interest Charges	8,214
Property Tax	36,229
Sales Tax	0[1]
Depreciation	29,370
Average Daily Attendance	273

1 None of the schools reported purchases of instructional supplies.

APPENDIX GG

SUMMARY STATISTICS 14 INDEPENDENT AND SEPARATE SCHOOLS

Item	Mean
Administrative Salaries	$ 13,172
Administrative Total	28,620
Plant Operation and Maintenance	72,954
Instructional Salaries	91,390
Instructional Supplies	4,292
Interest on Temporary Borrowing	1,999
Long Term Interest Charges	8,214
Property Tax	36,229
Sales Tax	0[1]
Depreciation	29,370
Average Daily Attendance	273

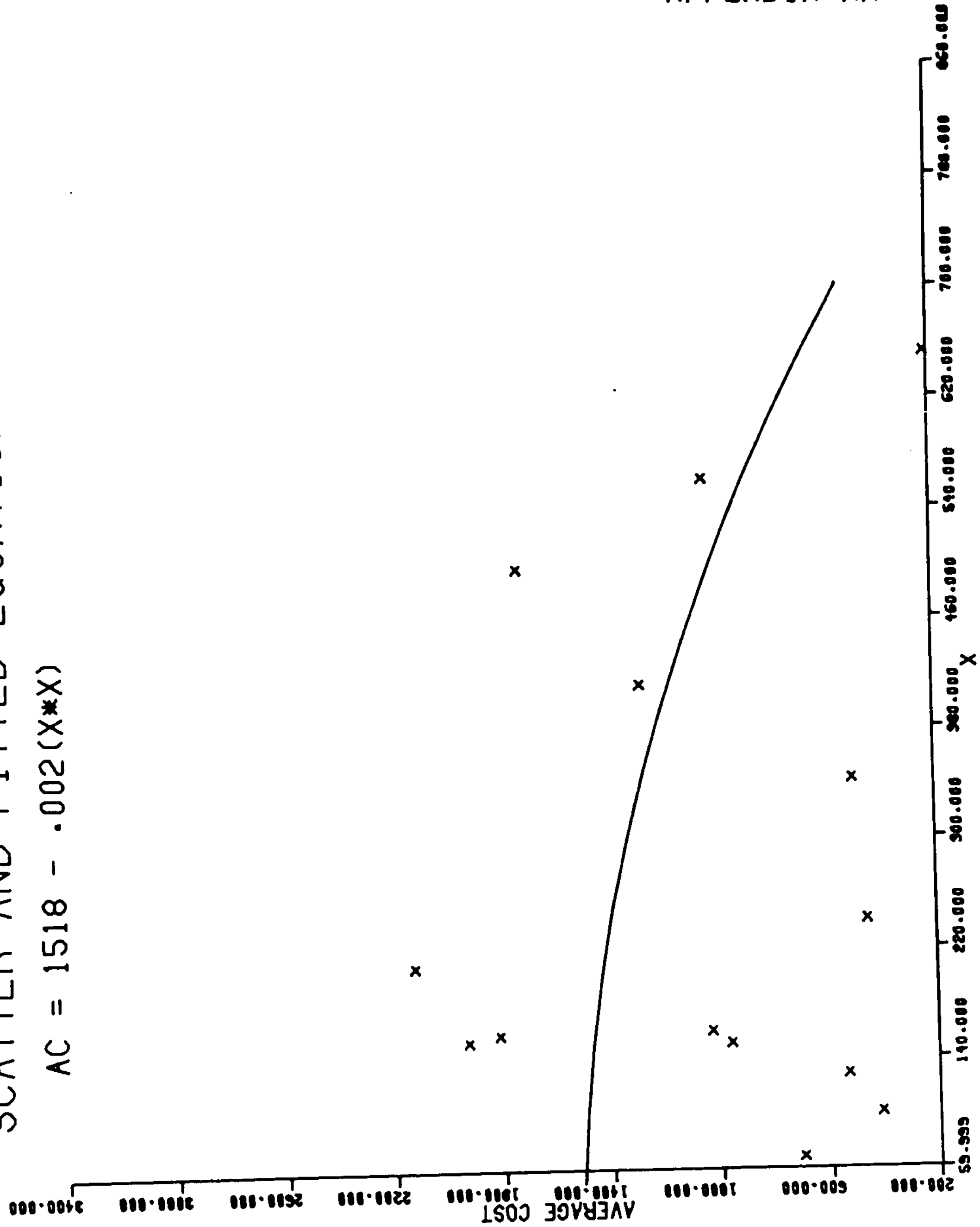
1 None of the schools reported purchases of instructional supplies.

APPENDIX HH

INDEPENDENT AND SEPARATE SCHOOLS

SCATTER AND FITTED EQUATION - RAW DATA

AC = 1518 - .002(X*X)

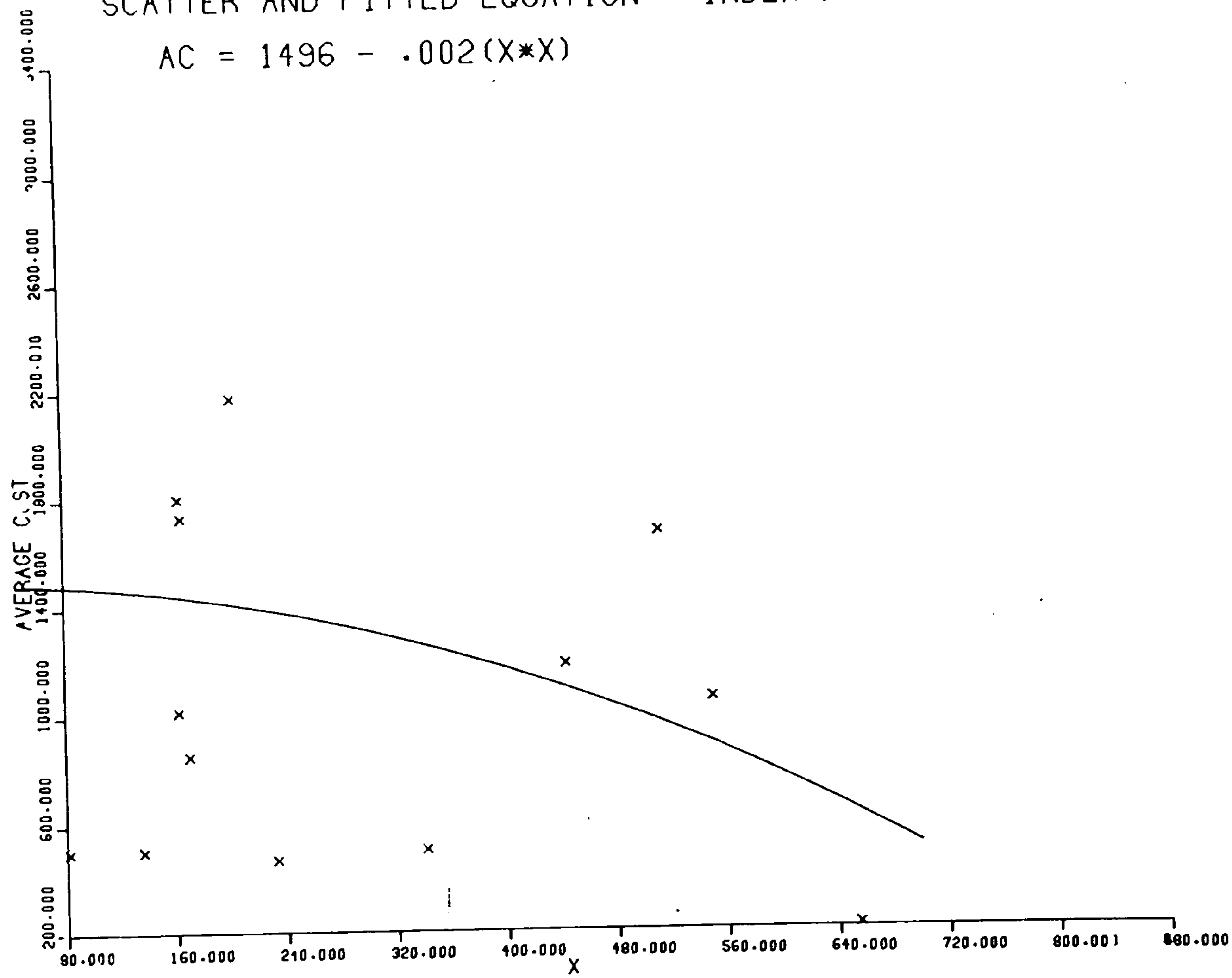


APPENDIX II

INDEPENDENT AND SEPARATE SCHOOLS

SCATTER AND FITTED EQUATION - INDEX A WEIGHTED DATA

$$AC = 1496 - .002(X*X)$$



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BIBLIOGRAPHY

Anderson, G. J. Input-Output Difference Among Secondary Schools. Unpublished M. A. Thesis. McGill -- MacDonald College, 1966.

Anderson, H. A. (Ed.) The High School in a New Era. Chicago, The University of Chicago Press, 1958.

Ayres, L. P. Laggards in Our Schools. New York, Russell-Sage Foundation, 1909.

Baird, N. E. Educational Finance and Administration for Ontario. Toronto, Ontario College of Education, 1952.

Benson, C. S. The Economics of Public Education. Boston, Houghton Mifflin, 1961.

_________. Perspectives on the Economics of Education. Boston, Houghton Mifflin, 1963.

Bowen, W. G. Economic Aspects of Education. Princeton, N. J., Princeton University, 1964.

Blaug, M. Economics of Education. Toronto, Pergamon Press, 1966.

Blitz, R. "The Nation's Educational Outlay," Economics of Higher Education, S. J. Muskin (Ed.). United States Department of Health, Education and Welfare, Bulletin, 1962, No. 5. Washington, 1962, pp. 147-73.

Brown, A. Changing School Districts in Canada. Toronto, The Ontario Institute for Studies in Education, 1968.

Boyle, G. J. The Cost of Education Index. Albany, New York (State) University, Division of Research, 1959.

Burke, A. V. Financing Public Schools in the United States. (Rev. Ed.) New York, Harper & Bros., 1956.

Burkhead, J. Public School Finance: Economics and Politics. Syracuse, N. Y., Syracuse University Press, 1964.

________, T. G. Fox and J. W. Holland. Input and Output in Large City Schools. Syracuse, N. Y., Syracuse University Press, 1967.

Canadian Teachers Federation. The Piper and the Tune. Record of the 1967 Conference on Educational Finance, Ottawa, 1967.

Castetter, W. B. and L. Ovsiew. Budgeting for Better Schools. Englewood Cliffs, N. J., Prentice-Hall, 1960.

Cheal, J. E. Investment in Canadian Youth. Toronto, The MacMillan Company of Canada Limited, 1963.

________. "Counting the Cost of Canada's Denominational School Systems," School Quarterly, Spring, 1964, pp. 100-115.

Clark, H. F. Cost and Quality in Public Education. Syracuse, N. Y., Syracuse University Press, 1963.

Cooke, M. E. Academic and Industrial Efficiency. New York, N. Y., The Carnegie Foundation for the Advancement of Teaching, 1910.

Edding, Friedrich. Methods of Analysing Educational Outlay. Paris, UNESCO, 1966.

Ellis, J. W. An Investigation of the Cost Function for Kansas School Districts, 1965-66. Unpublished M. A. Thesis, Department of Economics, Wichita State University, June, 1967.

Firman, W. D. "The Relationship of Cost to Quality in Education," Long Range Planning in School Finance. Based upon the Proceedings of the Sixth National School Finance Conference, St. Louis, April 7-9, 1963, pp. 101-110.

Gauerke, W. E. and J. R. Childress (Eds.). The Theory and Practice of School Finance. Chicago, Rand-McNally, 1967.

Goodman, S. M. The Assessment of School Quality. The University of the State of New York (State Education Department), 1959.

________. Procedures in School Quality Evaluation. Albany, The University of the State of New York, 1959.

Hanson, N. W. "Economy of Scale as a Cost Factor in Financing Public Schools," National Tax Journal, March, 1964, pp. 92-95.

Hirsch, W. Z. "Analysis of Rising Cost in Public Education," Joint Economic Committee, Congress of the United States, Study Paper #4, 89th Congress, 1st Session, Washington, United States Government Printing Office, 1959, pp. 1-43.

________. "Determinants of Public Education Expenditures," National Tax Journal, March, 1960, pp. 29-40.

Johns, R. L. and E. L. Morphet. Financing the Public Schools. Englewood Cliffs, N. J., Prentice-Hall, Inc., 1960.

________ and E. L. Morphet (Eds.). Problems and Issues in Public School Finance. New York, Bureau of Publications, Columbia University, 1952.

Johnson, M. and E. Scriven. "Class Size and Achievement Gains," School Quarterly, Autumn, 1967. pp. 300-310.

Johnston, J. Statistical Cost Analysis. New York, McGraw-Hill Book Company, 1963.

Kershaw, J. A. and R. N. McKean. Systems Analysis and Education. Santa Monica, RAND Corporation Research Memorandum RM-2473-FF, October, 1959.

Koos, L. V. Private and Public Secondary Education--A Comparative Study. Chicago, University of Chicago Press, 1931.

McLure, W. P. Educational Cost Analysis. Bureau of Educational Research, College of Education, University of Illinois, March, 1957.

McMillan, R. T. "Comparison of Farm Housing Indexes for Oklahoma," Social Forces, October'45 - May'46, pp. 174-180.

Miner, J. Social and Economic Factors in Spending for Public Education. Syracuse, N. Y. Syracuse University Press, 1963.

Moffatt, H. P. Educational Finance in Canada. Toronto, W. J. Gage Limited, 1957.

Mort, P. R., C. A. Newell and W. S. Vincent. The Growing Edge: An Instrument for Measuring the Adaptability of School Systems. New York, New York Metropolitan School Study Council, Teachers College, Columbia University, 1946.

National Educational Association. The Principals Look at the Schools. Washington, National Educational Association, 1962.

Netherlands Central Bureau of Statistics. Statistical Investigations on Education and Leisure (2) -- Size and Cost of Secondary Grammer Schools. The Hague, 1965.

Ontario Department of Education. Report of the Minister, 1964. Toronto, 1965.

________. Report of the Minister, 1967. Toronto, 1968.

________. *The Reorganization of School Jurisdictions in The Province of Ontario.* Toronto, 1968.

Raymond, R. "Determinants of the Quality of Primary and Secondary Public Education in West Virginia," *The Journal of Human Resources*, Volume III, No. 4, pp. 450-470.

Rideout, E. B. and S. Najat. *City School District Reorganization.* Toronto, Ontario Institute for Studies in Education, 1965.

Riew, J. "Economies of Scale in High School Operation," *Review of Economics and Statistics*, August, 1966, pp. 280-287.

Robinson, E.A.G. and J. E. Vaizey (Eds.). *The Economics of Education.* New York, St. Martin's Press, 1966.

Ross, Donald H. (Ed.). *Administration for Adaptability.* New York, New York Metropolitan School Study Council, 1958.

The University of the State of New York, Division of Research. *Procedures in School Quality Evaluation.* Albany, January, 1961.

Thomas, J. A. "Efficiency in Education: An Empirical Study," *Administrator's Notebook.* October, 1962.

Vaizey, J. *The Costs of Education.* London, Allen and Unwin, 1958.

________. *The Economics of Education.* London, Faber & Faber, 1962.

________. The Control of Education. London, Faber & Faber, 1963.

VanDenHaag, E. *Education as an Industry.* New York, A. M. Kelley, 1956.

Walters, A. A. "Production and Cost Functions," *Econometrica*, January-April, 1963, pp. 1-66.

Weisbrod, B. A. "Education and Investment in Human Capital," Journal of Political Economy, Supplement, October, 1962, pp. 106-124.

West, E. G. "Private Versus Public Education--A Classical Economic Dispute," Journal of Political Economy, October, 1964, pp. 465-475.

________. Education and the State. London, The Institute of Economic Affairs, 1965.

Woodhall, M. and M. Blaug. "Productivity Trends in British Education 1938-62," Minerva, Summer, 1965, pp. 483-498.